

Enhancing Students' Understanding of Operation in Integers through Joyful Learning with GeoGebra™ Applet

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Abstract

This paper demonstrates a teacher-as-researcher intervention study by utilizing GeoGebra™ activities in the teaching and learning of arithmetical operations with integers. The researcher evaluated how the applet can elevate students understanding of the concepts of arithmetic integer operations. The data was collected through recordings of students' worksheets and communications in a virtual classroom. The teacher monitored students' progress and provided immediate suggestions and assistance via several methods. As a result, students could communicate and interact with each other and the teacher through the applet. Students were working on online activities as it happened during the COVID-19 pandemic period. A joyful and meaningful context was implemented in the lessons. The results were tremendously significant and hopefully will inspire readers and teachers.

Keywords: GeoGebra™, joyful learning, operations with integers, teacher-as-researcher.

Introduction

Historically, integers were demonstrated with numbers by employing a positive sign (+), zero (0) and numbers utilizing a negative sign (-). Many studies have attempted to examine which models and real-world contexts would be more helpful in enhancing students' comprehension of integer operations. The study illustrated in this article struggles to contribute one of the models and contexts for teaching integers. It demonstrates the result of a case study organized over two weeks in July-August 2021 during the COVID-19 pandemic period. The researcher was concerned with integer operations and some of the properties of arithmetic operations (addition, subtraction, multiplication, and division) by employing a GeoGebra™ applet. Students followed the instructions in the GeoGebra™ applet by carrying some tools in GeoGebra™ and performing a joyful experience while playing with their gadgets.

The researcher concentrated on producing a joyful learning process in the classroom. Joyful learning in mathematics can be performed if the teacher enables students to explore various mathematic concepts by employing numerous methods and techniques in accordance with students' interests. It is associated with a current issue in mathematics education around constructivism. Haylock and Thangata (2007) explain that constructivism concerns attention on the pupil's learning rather than on the teacher's teaching. The challenge for a teacher is to design experiences which engage students and foster generating mathematical meaning which can be implemented or transferred to other situations. Constructivism as a theory of learning is more than learning by performing or experimental learning. Still, in the frame of constructivism, students can manipulate mathematical materials in their method in the learning process, and they are also able to convert their understanding into their representations or other contexts.

The implication of the statements above is that mathematics teachers must enhance both competencies, first, in the pedagogy of teaching mathematics associated with how teachers

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employ various strategies in the classroom to facilitate students to learn mathematics meaningfully and joyfully. And secondly, the subject matter related to the knowledge of the structure of mathematics and how the mathematics teacher correlated the concepts and made students willing to understand the concepts in mathematics. Students learn mathematics through the experiences that teachers provide. Teachers should select and utilize one or more suitable methods or strategies; hence, students can study mathematics joyfully and meaningfully.

The utilization of innovative technology in mathematics education is escalating year by year. Some online and offline resources can be employed to enhance students' interest in the learning process. Technology transforms the teaching and learning process into a more active, joyful, and meaningful event. Students can observe and unveil various mathematical concepts from multiple resources and understandings. Teaching mathematical concepts could incorporate a variety of learning approaches between online or offline media to enhance students' participation in learning Mathematics. Based on the pandemic era, online media is appropriate for students through online software they access from their homes. Some of the Mathematics software available in the marketplace can assist the teacher in easily sharing the material or the activity with the students. Although, it depends on the teacher to organize the existing materials and media around them without the allocated extra preparation time to design and produce the latest teaching media (Arbain & Shukor, 2015).

GeoGebra™ is a free dynamic mathematics software invented by Markus Hohenwarter in 2001 at the University of Salzburg, Austria. GeoGebra™ can be implemented in classrooms for interactive and enjoyable mathematics teaching and learning processes from primary grade to high grade. GeoGebra™ incorporates various mathematical concepts such as algebra, geometry, statistics and calculus into an attractive package. The students can access the official GeoGebra™ website at <http://www.geogebra.org>. We can also download the latest version of the software from the website.

Furthermore, the online version also facilitates the same facility (Kushwaha, Chaurasia, & Singhal, 2014). The students can also produce an account on the GeoGebra™ website. Thus, they do not have to download it to their personal computer; it can be accessed from your browser. Numerous researchers recommend that GeoGebra™ software be utilized to promote the invention and experimentation of general mathematics teaching methods into joyful activities suitable for student interest. Various researchers suggest that teachers learn to use the latest technology in their teaching and learning process to integrate technology effectively (Ruthven, Hennessy, & Brindley, 2004).

GeoGebra™ was designed mainly for educational purposes. It can assist students in employing experimentation on a problem and their investigation in learning mathematics, both in the classroom and at home. Students can enhance their competencies in joyful ways (Diković, 2009). The benefit of utilizing GeoGebra™ are: it is uncomplicated to use; users can associate dynamic applets with producing activities; they personalize their activity in their classroom, and students can access the activity through their gadget from home. It is tremendously advantageous to assist teachers in formulating an online activity for their students comprising mathematical concepts, mainly during the pandemic period.

Based on the several benefits mentioned above, this researcher developed a task for GeoGebra™ Classroom. In four meetings, students took part in the activities that were planned. The exercises were created and refined in an environment of joyful learning. For the teaching

and learning process to take place joyously and meaningfully, a robust relationship between the instructor and student must be generated without coercion or stress. Additionally, there must be independence and a democratic environment. One method of teaching and learning that assists students in obtaining an understanding of and proficiency with mathematical ideas is joyful learning. (Asmani, 2011). The joyful scaffolding in teaching mathematics can encourage better learning. The student's cognitive development was enhanced, knowledge achievements were expanded, and psychomotor skills were livened up and multiplied through enjoyable mathematics learning activities. Fun learning also developed students' interest in engaging with activities and maximizing their learning (Yabo, 2020).

Through joyful learning, the researcher selected GeoGebra™ in teaching and learning arithmetic operations (addition, subtraction, multiplication, and division) with integers. The objective of the study is also to advance teacher's perspectives on the successful utilization of technology while concerning teacher's pedagogical values, adopting a naturalistic approach to generate more ideas, and performed in an educational system in which the effectiveness of online classroom and the utilization of computers or smartphone has been under-researched in Indonesia.

Methodology

In an ordinary school classroom, a teacher continuously investigates the students in the lesson during the progression of teaching and learning. Therefore, every teacher frequently receives much information during the instruction process. How the teacher reacts to this information incorporates a process of discerning what is crucial and what can be discarded. The teacher performs upon this information in several practices. However, the collection of this information is frequently not systematic and is generally not documented. In contrast, the teacher-as-researcher is a research design regarded with the systematic collection of data to examine and enhance the effectiveness of a program, practice or policy.

“The teacher-as-researcher movement lay in a paradigm shift that focused on teachers as knowers and thinkers. This shift grounded theory in practice and insisted that knowledge derived from the research was necessarily personal.” (White, Jaworski, Agudelo-Valderrama, & Gooya, 2013, p. 399).

This paper demonstrates a teacher-as-researcher design and an intervention research study. As the teacher-as-researcher is a qualitative study, issues of bias must be concerned. The Hawthorne effect is a reaction bias in which people alter a behavior pattern in response to being aware of being watched. It is suggested that the Hawthorne effect is irrelevant to this study because the students in this study were accustomed to being observed by a teacher, and the situation was familiar to them. The study also encompasses an intervention that entails the passionate teacher's introduction of a new technological tool. The issues that must be addressed are whether the same outcomes would be possible if the teacher and students were not as excited about performing the technology. It was no longer a fresh experience for the students. This issue will be elaborated on later in this paper.

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This study aims to identify whether the GeoGebra™ applet can be employed to enhance students' understanding of the arithmetic operation of integers. The activity is accessed from the following address: <https://www.GeoGebra.org/classroom/tw82hm7z>.

The researcher implemented the utilization of the applet in six classrooms from 7A – 7F in SMP (Junior High School) 1 Petanahan in this study. The students from these classes participated in this study. Most students could join the online activity, as each student possessed the proper smartphone in their home. The participants were evaluated during various performances per the classroom teacher and observation. The researcher collected data through multiple methods such as screen recording in the virtual classroom, the WhatsApp group, YouTube™ streaming, and student online worksheets.

Result and Discussion

The researcher conducted the study during the COVID-19 pandemic period. The students utilized their gadgets to operate GeoGebra™ Classroom from their own homes. The researcher monitored students through her laptop to identify their progress and to communicate via live YouTube™ Streaming also in WhatsApp group in the virtual classroom of Junior High School 1 Petanahan. Meanwhile, students performed on GeoGebra™ Classroom, and their activity was recorded. The recording could be employed as an assessment item because it encompassed the process of how they solved the tasks.

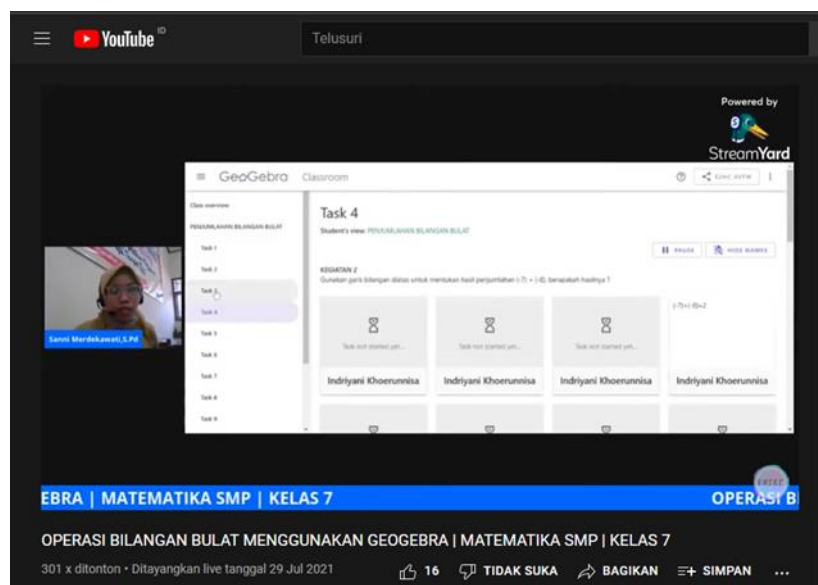


Figure 1. The researcher monitors student activities through live YouTube™

The activity comprised four parts: addition, subtraction, multiplication, and division. Each piece possesses the goal that students identify some properties of arithmetical operation by experimenting on the line number. Students move the button to determine the answer and type down their answer in the box provided. They can also produce a conclusion and share their ideas in the box. While the students work with the activity, the teacher can monitor them through the GeoGebra™ website. Some students could access the activity easily, but others could not attain access due to: incompatible gadgets, signal difficulties, and other technical

problems. Meanwhile, students were implementing their activities, the researcher presented their progress in the live YouTube™, assisted those encountering difficulties or technical problems, motivated and encouraged the students to attempt and complete the tasks and allowed them to share their ideas.

Student Experiments

Working with GeoGebra™ can turn the activities of the learning process in Mathematics into a joyful learning process. They can explore the activity and move the object in the applet by themselves. The researcher believes that students can produce their own fun experience in mathematics by utilizing GeoGebra™. Students generate trial and error experiments until they can identify their "aha!" moments and determine the correct answer. The teacher merely ensures they can access the activity, perform it with happiness and joy, and share their happiness with the other; thus, joyful learning can be established in the classroom by themselves.

In joyful learning, students require to be more active as a facilitator. It can be implemented in 4 steps: experiment, interactive, communication and reflection (Asmani, 2011). The second figure reveals the experience of students who can investigate the activity, move the button and discover the correct answer on addition. It is a part of the experiment step.

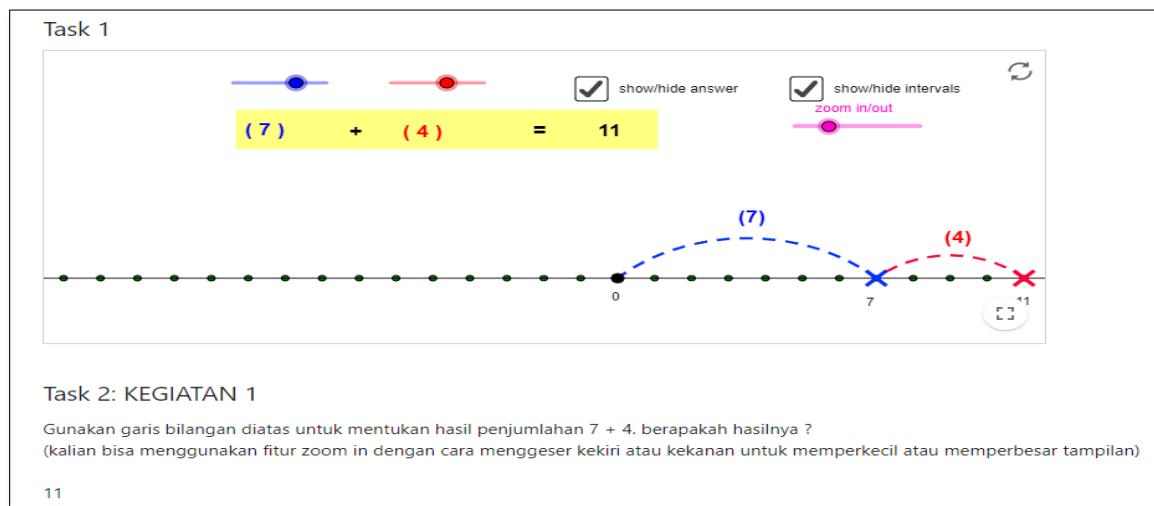


Figure 2. A student's work in part one (addition) of a worksheet.

It can be perceived that s/he could correctly move the red and blue buttons and determine the correct addition on Activity 1. Another student did not correctly move the red and blue buttons, but s/he could generate the correct answer on the activity. Occasionally, students encounter a technical problem with their gadgets, but they still produce an effort to complete the activity and determine the right solution. It is presented in Figure 3.

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Task 1

Task 2: KEGIATAN 1

Gunakan garis bilangan diatas untuk menentukan hasil penjumlahan $7 + 4$. berapakah hasilnya ?
(kalian bisa menggunakan fitur zoom in dengan cara menggeser kekiri atau kekanan untuk memperkecil atau memperbesar tampilan)

$7+4=11$

Figure 3. Another student's work in part one/addition of the worksheet.

They also examined subtraction. Some students could answer the activity correctly but did not discover the conclusion, and others could generate the conclusion. It is displayed in Figure 4. From this situation, the teacher can encourage the student by instructing them to produce a conclusion of the activity.

Task 12: KEGIATAN 7

Gunakan garis bilangan diatas untuk menentukan hasil pengurangan $12 - 4$. berapakah hasilnya ?
(kalian bisa menggunakan fitur zoom in dengan cara menggeser kekiri atau kekanan untuk memperkecil atau memperbesar tampilan)

8

Task 14: KEGIATAN 8

Gunakan garis bilangan diatas untuk menentukan hasil pengurangan $4 - 12$. berapakah hasilnya ?
apakah sama antara hasil operasi pengurangan $12 - 4$ dengan $4 - 12$? jelaskan jawabanmu

-8

Figure 4. Students' worksheets in parts A and B of subtraction, but s/he did not generate the conclusion.

Students' Interaction and Communication

The part of interaction and communication can be discovered in the chat box of the virtual classroom through the WhatsApp group. The illustration of their conversation is displayed below:

- Student A : "Mam, I cannot move the red button."
Student B : "Just try it! Make sure the loading process is almost completed."
Researcher : "Perhaps, you can change your gadget into landscape mode."
Student A : "Thank you, mam; I can see the buttons clearly and move them well."
Student B : "Mam, can I try all the activities now? It is fun, and I am eager to get into the next step."
Researcher : "Wow, definitely, you can perform it right away."
Student C : "Mam, I want to try it again. I think I have a new idea for my activity before."
Researcher : "You can perform it as much as you can, dear."
Student C : "Thank you, mam. I love this game. I can start it all over again and learn how to add negative and positive numbers with a number line. It also displays the right answer, so I can check it myself."
Researcher : "Good job, students. Do you like the activities?"
Student B : "I like it. It is fun. I want to do it more."
Researcher : "Well done, but unfortunately, the time is over. You have to learn another subject now."
Student A : "The time seems so fast."
Researcher : "Don't worry. You can try it again next week and also try it outside school hours."
All the students : "Thank you, mam."

We can infer from this dialogue that students enjoyed the activities because playing with the applet was enjoyable. They can restart the activity if they possess an idea for a new worksheet. They can move the buttons, manipulate the number line, and independently develop the concept and fundamentals of addition, subtraction, multiplication, and division. They are intrigued to perform the activity. Students can be encouraged by interacting with others. Therefore, we may conclude that the GeoGebra™ exercise can be utilized to inspire students and make learning more enjoyable. The researcher also kept an eye on all the students because some were unable to complete the task because of a weak signal or a device that was incompatible. The researcher encouraged them to attempt at a different time and assisted them with the trouble.

On the other hand, some students possessed high performance; they could perform all the activities correctly and on time and required more challenging activities. Therefore, the researcher prepared another series of worksheets for the students who completed the activity; thus, they enhanced their ability and allowed them to experience more challenging tasks. One of the activities which could be conducted by high-performing students who generated a conclusion can be presented in Figure 5.

Task 7: KEGIATAN 4
Gunakan garis bilangan diatas untuk mentukan hasil penjumlahan $(-14) + 6$. berapakah hasilnya ?
- 8

Task 9: KEGIATAN 5
Gunakan garis bilangan diatas untuk mentukan hasil penjumlahan $6 + (-14)$. berapakah hasilnya ?
- 8

Task 10: KEGIATAN 6
Pada kegiatan 4 dan 5. apakah kalian menemukan hasil yang sama ? jelaskan sifat apakah itu ?
hasil kegiatan 4 dan 5 hasilnya sama yaitu -8.
termasuk sifat komulatif karena hasil yang didapatkan sama hanya terjadi pertukaran tempat bilangan bulat

Figure 5. One of the student's worksheets with high performance.

Student Reflection and Conclusion

In Activity 3, we can discover various student answers from activities 1 and 2. It is demonstrated in Figure 6.

<p>Task 5: KEGIATAN 3</p> <p>Buatlah kesimpulan dari kedua kegiatan diatas, tentang penjumlahan dua bilangan bulat positif dan dua bilangan negatif.</p> <p>Bilangan positif semakin ke kanan semakin besar bilangan negatif semakin ke kiri semakin kecil</p>
<p>Task 5: KEGIATAN 3</p> <p>Buatlah kesimpulan dari kedua kegiatan diatas, tentang penjumlahan dua bilangan bulat positif dan dua bilangan negatif.</p> <p>Bilangan positif di tambah bilangan positif sama dengan positif, bilangan negatif di tambah negatif sama dengan negatif</p>

Figure 6. Variety of answers from different students.

The researcher could observe how the students produced the conclusion about addition. The researcher could also confirm their conclusion by administering it in the WhatsApp group. The researcher could also check the student who can pass all the activities and allow them to share their ideas in the virtual classroom. The representation of the students who could give the activities and also generate a conclusion is illustrated in Figure 7.

Task 35: KEGIATAN 18

Untuk sembarang bilangan bulat a dan b . Apakah $a : b = b : a$? jelaskan jawabanmu

Tidak
karena hasilnya tidak sama contoh $-20 : 5$ tidak sama dengan $5 : (-20)$

Task 36: KEGIATAN 19

a. pembagian sembarang bilangan bulat positif akan menghasilkan bilangan

Check all that apply

positif
 negatif

b. pembagian sembarang bilangan bulat positif dan negatif akan menghasilkan bilangan

Check all that apply

positif
 negatif

Task 39: KEGIATAN 20

Dapatkan kamu menyimpulkan konsep atau sifat operasi pembagian pada bilangan bulat? jelaskan pendapatmu

- Hasil bagi dua bilangan bulat yang mempunyai tanda sama selalu positif.
- Hasil bagi dua bilangan bulat yang mempunyai tanda berbeda selalu negatif.
- Tidak bersifat komutatif
- tidak bersifat asosiatif
- Tidak bersifat distributif

Figure 7. Students' worksheets who completed all the activities and wrote a conclusion involving division.

Even though the activities are quite simple, it was crucial to assist the students in comprehending the concepts of arithmetical operations and to prepare them for the next chapter of mathematical concepts in algebra, geometry and others. Most students could engage with GeoGebra™ activities, complete the tasks with happiness and joy, and develop their conceptual grasp of addition, subtraction, multiplication, and division by implementing integers on their own. Furthermore, students encouraged one another and created positive memories of the mathematics learning experience by sharing their joy. The ideas of commutative and associative in addition and multiplication were also accessible to the students. This study revealed that GeoGebra™, with its action in positive and negative numbers in integers, can be a robust and advantageous instrument for visualization and stimulation of the number line. It might demonstrate how the process goes. Moreover, students could improvise with the applet.

Conclusion

The researcher acknowledges that the results may be subject to the biases presented in the methodology section encompassing the unassessed contamination effects of the novelty of the technology and the teacher's enthusiasm. Within these qualifications, this paper concludes that the activities in the GeoGebra™ applet corroborated students to develop their understanding of arithmetical operations of integers. Data presented that students could begin the exercises and keep attempting the activity when they encountered difficulties or obtained some latest ideas. The GeoGebra™ activities encouraged students to experiment with the mathematical concepts in an interactive and communicative environment with other students and the teacher and allowed them to reflect and produce conclusions at the end of the activities. It was demonstrated that GeoGebra™ assisted students in enhancing their intuitive feeling and in visualizing the

mathematics processes in the operations of integers. With the aid of GeoGebra™, students could investigate various integer-related characteristics and make links between symbolic and visual representations of mathematics. Students could easily redo the assignments multiple times to further their understanding. The teacher also could monitor the status of each student's activity from the GeoGebra™ website.

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