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Problem-Solving Skills and Productive Struggle of Students in Solving Mathematical Problems in Elementary School

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Abstract

The research is motivated by the need to gain a deeper understanding of students' mathematical problem-solving abilities, particularly in algebra, and to identify and address the productive struggles the encounter while solving problems. This research aims to analyze students' mathematical problem solving abilities in solving algebra problems and identify students' productive struggles in solving these problems. This research was carried out using a qualitative approach with descriptive methods and instruments used in this research are problem solving ability tests, questionnaires that have been validated by education experts, and interview guides. Researchers selected students based on their level of productive struggle, namely high, medium, and low. The result revealed that students who have high productive struggle can answer the three questions correctly according to the problem solving stages, starting from the understanding stage, transformation stage, process skills stage, and conclusion stage. Students who have moderate productive fighting power can answer the three questions only up to the understanding and transformation stage. At the process skills stage, students show errors in calculating and need teacher motivation to continue their struggle in solving mathematical problems. Students who have low productive fighting power can answer questions only up to the understanding stage, limited to writing back what they know and ask. In this category, students need intervention assistance from researchers to encourage their productive struggles. This assistance does not eliminate students' opportunities to think actively, on the contrary, through this method students are required to interpret their knowledge.

Keywords: *problem solving ability; productive struggle*

Introduction

21st-century mathematics education emphasizes the importance of developing six skills that include critical thinking and problem solving, communication, collaboration, creativity and innovation, character education, and citizenship (Kennedy, T. J., & Sundberg, C. W, 2020; González-Pérez, L. I., & Ramírez-Montoya, M. S, 2022). These skills must be integrated and implemented in mathematics education at schools. A teacher, as an intermediary of information to students, plays a crucial role in ensuring the success of these integrations. This aligns with the National Council of Teachers of Mathematics (NCTM, 2017), which established five standards of mathematical competencies to realize the objectives in mathematics learning: problem-solving, reasoning, communication, connection, and representation.

Mathematics education in schools aims not only to understand the taught material but primarily to ensure that students possess the abilities to reason, communicate, represent, and solve problems, all of which are components of mathematical thinking. (Szabo, Z. K., Körtesi, P., Guncaga, J., Szabo, D., & Neag, R, 2020; Care, E., Kim, H., Vista, A., & Anderson, K, 2018; Ariawan & Nufus, 2017). Mathematical thinking is the process of developing a mathematical perspective, appreciating the process of mathematization, having a strong desire to apply it, and developing competencies while equipping oneself with all necessary tools, then simultaneously using these tools to understand the structure of mathematical comprehension (Schoenfeld dalam Fajri, 2017). Mathematical thinking is defined as a dynamic process that allows us to enhance the complexity of ideas we encounter, which expands our understanding (Primasatya, 2016).

There are four concepts related to mathematical thinking: (1) mathematical abilities; (2) mathematical skills; (3) performing mathematical processes (doing mathematics); (4) mathematical tasks (Stein, M. K., & Lane, S, 1996; Henningsen, M., & Stein, M. K, 1997; Fajri, 2017). From these, each can be identified as an assumption that mathematical thinking is the implementation in carrying out activities or mathematical processes (doing math) or mathematical tasks. The ability to use mathematical thinking in problem-solving is one of the most fundamental goals of mathematics teaching, yet it is also one of the most challenging to comprehend (Phonapichat, P., Wongwanich, S., & Sujiva, S, 2014; Pongsakdi, N., Kajamies, A., Veermans, K., Lertola, K., Vauras, M., & Lehtinen, E, 2020).

The ultimate goal of teaching is for students to be able to conduct their mathematical investigations and to recognize that the mathematical themes they have learned can be applied in real-world situations. In the words of the mathematician Halmos (1980), problem-solving is "the heart of mathematics." Therefore, the ability to think mathematically represents an accumulation of the concept of thinking mathematically, indicating the development of capabilities: (1) mathematical understanding; (2) mathematical problem-solving; (3) mathematical reasoning; (4) mathematical connections; (5) mathematical communication. Individuals who possess mathematical thinking skills naturally have mathematical competence within them. According to Kilpatrick (2001: 103), mathematical competence consists of five types: Conceptual understanding; Procedural fluency; Strategic competency; Adaptive reasoning; and Productive disposition. These five strands of mathematical competence are not separate but interwoven into a single competence representing different aspects of something complex. Mathematical competence is not merely an “innate” ability of students, but a combination of knowledge, skills, abilities, and beliefs acquired by students with the help of teachers, the curriculum, and a dependable learning environment (classroom). The observations have implications for how students acquire their mathematical abilities, how teachers develop these abilities in their students, and how teachers are educated to achieve these goals.

In elementary school mathematics education, it is crucial for students to optimally possess all five types of mathematical competencies to achieve the objectives of mathematics learning. This aligns with the NCTM (2000) goal of mathematics education, which is directed towards mathematical problem-solving abilities. Kemdikbud (2013) states that mathematical problems serve as a means to develop competencies in logical, critical, and creative thinking, as well as the ability to choose and apply strategies in problem-solving. This underscores that strategic competence, as one of the mathematical competencies, is vital for students to possess to solve mathematical problems effectively. Thus, it can be said that the process of solving mathematical problems significantly requires strategic competence as a solution.

Strategic competence is influenced by the ability to understand mathematical concepts (Kilpatrick, 2001). This implies that when students attempt to apply problem-solving strategies, a prerequisite is that they must have a robust understanding of mathematical concepts and be familiar with various types of mathematical problems. According to Balai Pustaka (1995), a problem is a question or something that needs to be resolved (solved), meaning that typically, a problem takes the form of a question that needs to be answered and solved. This view is further reinforced by Suherman (2003) who stated that mathematical problems contain situations that compel a person to want to resolve them.

However, to solve these problems, students must possess relevant strategic competencies in problem-solving. Students also need time to think of appropriate ways or strategies to solve these problems. Therefore, solving problems necessarily involves experiences in tackling various problems previously, meaning that students cannot immediately know the correct way to solve a problem without prior experience (Jonassen, D., Strobel, J., & Lee, C. B, 2006)

Problems can be categorized into two types: routine problems and non-routine problems (Chong, M. S. F., Shahrill, M., Putri, R. I. I., & Zulkardi, Z, 2018; Suherman, 2003). Routine problems are those that can be solved by applying mathematical procedures that are the same or similar to the material recently learned. On the other hand, non-routine problems are those that cannot be solved by using procedures that are identical or similar to the material recently studied by students in class. According to Polya (1973), mathematical problems are divided into routine and non-routine problems. Routine problems involve the application of routine calculations, while non-routine problems require solutions that demand creative thinking and the application of certain heuristic strategies to understand the problem situation and find ways to solve it. Therefore, this research utilizes non-routine contextual problems that require appropriate strategies. The process of solving these problems aligns with mathematical concepts and subsequently leads to the correct final solution (Kuo, E., Hull, M. M., Gupta, A., & Elby, A, 2013; Abdelshiheed, M., K. Jacobs, J., & K. D'Mello, S, 2024; Elia, 2009).

Polya (1973) proposed steps for mathematical problem-solving, which include understanding the problem, devising a plan for solving the problem, carrying out the plan, and reviewing the solution. Through these steps, students are expected to solve the mathematical problems they encounter. However, many students still make mistakes when solving mathematical problems. In this regard, Newman (1977) classified the types of errors students make in solving problem-solving tasks, including reading errors, comprehension errors, transformation errors, process skill errors, and encoding errors.

Reading errors occur when students cannot understand keywords or symbols in a problem (Vaughn, S., Boardman, A., & Klingner, J. K, 2024). Comprehension errors occur when students can read the information in the problem but do not understand the intent of the question. Transformation errors occur when students have understood the problem but cannot identify the strategy needed to solve it. Process skill errors occur when students can identify the strategy to be used but do not understand how to employ it effectively. Encoding errors occur when students fail to identify the problem and gather all necessary information to solve it (Newman, 1977).

Based on these insights, it can be concluded that providing problem-based tasks can certainly help students develop their strategic competencies, especially in solving mathematical problems. Therefore, to enhance students' mathematical problem-solving abilities, it is essential to formulate non-routine problems that require solutions through creative thinking and the application of specific heuristic strategies to understand the problem situation and find ways to solve it. It is also necessary to design supportive learning environments that allow students to engage in productive struggle. Furthermore, practical actions are needed to anticipate common errors that students make while solving mathematical problems. Thus, the focus of this study will be to analyze students' errors in using their mathematical problem-solving abilities from the stages of understanding, transformation, process skills, and encoding, with each process being interrelated, as well as to review the productive struggle of students in solving mathematical problems in elementary school as a fundamental basis for improving their mathematical problem-solving capabilities.

Methods

This study was conducted using a qualitative approach with descriptive methods to gain insights into students' abilities to solve mathematical problems and their productive struggle in facing difficulties. The subjects of this study were class V elementary school students in Bandung City, consisting of 32 students. The research instruments employed included a problem-solving ability test, questionnaires that had been validated by educational experts, and interview guidelines.

Data analysis was performed in three stages: examining the students' answer results, presenting the test and questionnaire data, and drawing conclusions from the research findings. To determine the percentage of each type of error in students' responses, the following formula was used:

$$P = \frac{n}{N} x\ 100\ \%$$

Where:
P: Percentage of error type
N: Total number of errors for each error type.

The researcher then conducted observations and interviews with students who experienced difficulties in solving mathematical problems. Interventions were used to facilitate students in maintaining their motivation amidst challenges and in producing accurate and productive solutions. Students were provided with support in this endeavor by offering help and guidance without compromising their opportunities to engage in the learning process (Brousseau 2002; NCTM 2017).

The categorization of students analyzed was grouped according to their level of productive struggle as follows:

Table 1. Categorization of Students' Productive Struggle

Category	Score
High	$x > 80$
Medium	$65 \leq x \leq 80$
Low	< 65

Indicators

The indicators used to assess mathematical problem-solving abilities are as follows:

Table 2. Indicators of Problem-Solving Ability

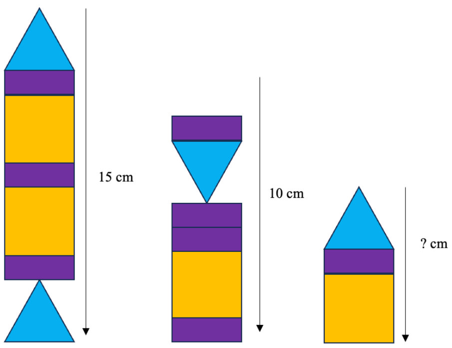
Indicator	Description
Understanding the Problem	Identifying known elements, questions, and sufficiency of elements
Planning the Solution	Creating a mathematical model
Solving the Problem as Planned	Applying strategies to solve the problem both within and outside mathematics, explaining and interpreting results
Checking the Solution	Revisiting the solution meaningfully according to the steps

Table 2 presents several key indicators that are emphasized for a deeper elaboration on how students solve mathematical problems. The researcher refers to the mathematical problem-solving steps implemented in the Singapore Curriculum, which facilitates students in overcoming difficulties in solving abstract mathematical problems.

Test Instruments

Three contextual mathematics problems were developed for this study.

Table 3. Test Items

No.	Item Description
1.	Zeeana’s height is 35 cm shorter than Fahrezan’s height. If the combined height of Zeeana and Fahrezan is 198 cm, determine the individual heights of Zeeana and Fahrezan and verify that their combined height equals 198 cm!
2.	A rectangle has a perimeter of 48 cm. Its width is 4 cm shorter than its length. Determine the length and width of the rectangle!
3.	Below are two combined flat shapes with different heights, consisting of squares and triangles.  What is the height of the shortest combined flat shape?

The problems were provided in descriptive form to allow the researcher to evaluate students' problem-solving abilities through their answers. Content and language structure validity were conducted in this study, with validation carried out by mathematics education experts. Items 1, 2, and 3 are designed with problem-solving ability indicators. In this research, students were given the freedom to choose concrete, pictorial, or abstract mathematical representation methods. This approach was intended to allow the researcher to observe students' tendencies in creating mathematical representations.

Results and Discussion

Problem-solving skills are fundamental in learning mathematics. In mathematics problems, problem-solving skills involve understanding the question, determining a solution strategy, and applying previously acquired knowledge to new, unfamiliar situations. Therefore, in the process of solving problems, prior knowledge must align with the problem at hand. Regardless of how extensive one's prior knowledge is, it cannot be used to solve a problem if it is not relevant. The problems used in this study are related to algebra for class V elementary school students.

The test was administered concerning algebra material. Students' answers were analyzed through four stages: understanding, transformation, process skills, and conclusion (Minarni, A. (2019; Compayan, L. U., & Dollete, M. L. A, 2019; Hwang, W. Y., Chen, N. S., Dung, J. J., & Yang, Y. L, 2007; Putra, 2018). These four stages are interrelated. If students can complete the first stage, they will be able to proceed to the second, third, and fourth stages. The following describes the stages that students went through in solving the problem-solving tasks.

1. Understanding Stage
   This stage aims to assess students' ability to comprehend the problem when converting information from the problem into a mathematical model. Data from the assessment showed that 81.25% of students answered correctly at the understanding stage, while the remaining answered incorrectly.
2. Transformation Stage
   This stage aims to assess students' ability to substitute the value of the first variable found in the mathematical equation or to define this ability as determining a problem-solving strategy. Data from the assessment revealed that 62.5% of students answered correctly at the transformation stage, while the remaining answered incorrectly.
3. Process Skills Stage
   This stage aims to evaluate students' ability to find the requested variable by constructing the previously created mathematical model. Data from the assessment indicated that 56.25% of students answered correctly at the process skills stage, while the remaining answered incorrectly.
4. Conclusion Stage
   This stage aims to assess students' ability to identify the problem, gather all relevant information, and conclude the solution. Students are expected to determine the answer based on the information obtained in the first, second, and third stages. Data from the assessment showed that 32.5% of students answered correctly at the conclusion stage, while the remaining answered incorrectly.

Here is the percentage of students who answered the problem-solving questions correctly and incorrectly.

Table 4. Percentage of Students' Correct and Incorrect Answers on Problem-Solving Questions

Stage	Correct	%	Incorrect	%
Understanding	26	81.25	6	18.75
Transformation	20	62.5	12	37.5
Process Skills	18	56.25	14	43.75
Conclusion	3	9.38	29	90.62
Overall Percentage	52.34		47.65	

Table 4 shows that out of 32 students, 3 students were able to answer the given problem-solving questions correctly. The percentage of students who answered all four aspects correctly is higher compared to those who answered incorrectly, at 52.34%. However, a significant number of students also answered incorrectly across all four aspects, totaling 47.65%. Students made the most errors in the conclusion stage, with 90.62%, followed by the process skills, transformation, and understanding stages.

Students performed best in the understanding stage, with a correct answer rate of 81.25%, followed by the transformation, process skills, and conclusion stages. This indicates that the stages of problem-solving are interconnected. If students do not understand the problem and lack the prerequisite skills or knowledge, they will struggle with subsequent stages. Students can solve problems effectively if they understand the problem from the start and possess the necessary prerequisite skills. These prerequisite skills relate to conceptual understanding. Therefore, students will be able to solve mathematical problems if they have the required conceptual understanding.

The following is problem-solving question number 1 given to students:

> Zeeana is 35 cm shorter than Fahrezan. If the combined height of Zeeana and Fahrezan is 198 cm, determine the individual heights of Zeeana and Fahrezan and verify that their combined height equals 198 cm!

The errors made by students while solving this problem were analyzed, and an example from the responses of students who made mistakes at each stage is presented.

Analysis of Errors in Students' Answers with Low Productive Struggle at the Understanding Stage

18.75% of students, or 6 students, made errors at the understanding stage. The following is an example of a student's response who made an error at the understanding stage and received a score of zero.

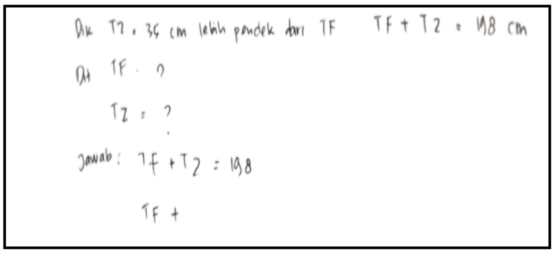


Figure 1. Incorrect Answer from a Student Making an Error at the Understanding Stage

In Figure 1, it can be seen that the student understood the question but did not grasp the meaning of the statement that Zeeana's height is 35 cm shorter than Fahrezan's. As a result, the student struggled to solve the problem. The student was unable to determine the value of x in the algebraic equation model and could not explain how to relate the two known variables.

On their answer sheet, the students merely wrote down the sum of the heights of Zeeana and Fahrezan, which was already given in the problem. The student struggled to establish the relationship between the heights of Zeeana and Fahrezan, preventing them from progressing to the next stage. The student was unable to create the expected mathematical model, leading to difficulties in solving the problem. According to the survey, most students dislike mathematics and tend to fear the subject because they find it challenging.

From the perspective of their productive struggle, the average score for students who made errors at the understanding stage was 55, indicating a low category. These students, when it comes to asking questions, are reluctant to ask their teacher or peers when confused. Regarding motivation, these students stop trying to solve the problem and do not attempt again. Furthermore, in terms of strategy, these students only check their answers when instructed by the teacher and lack initiative.

Of the three problems given, almost all were only completed up to the stage of noting what is known and asked. They need intervention and motivation to improve their productive struggle. This aligns with the thoughts of Pruner and Liljedahl (2020), who stated that if we want our students to think, we need to give them something to think about; something that not only requires thought but also encourages it. Therefore, the researcher provided actions in the form of verbal motivation and direct assistance to encourage students to try again and solve the problems.

Analysis of Errors in Students' Answers with Medium Productive Struggle at the Transformation Stage

37.5% of students, or 12 students, made errors at the transformation stage. Below is an example of a student's response who made an error at this stage, thus receiving a score of one.

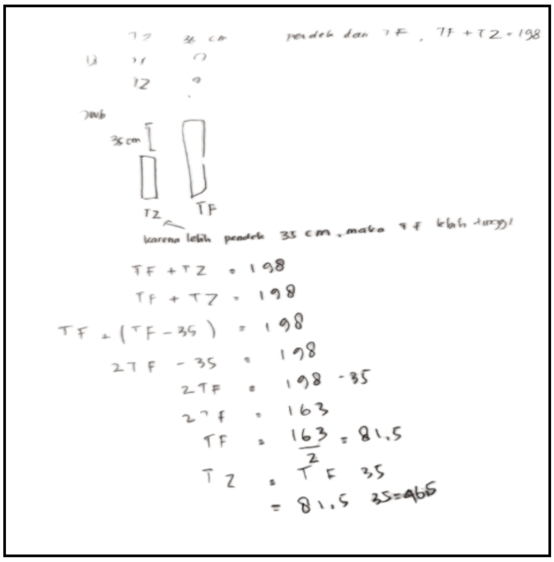


Figure 2. Incorrect Answer from a Student at the Transformation Stage

In Figure 2, it is evident that the student understood the question and grasped the meaning of the statement that Zeeana's height is 35 cm shorter than Fahrezan's, enabling them to create an algebraic equation as a mathematical model. The student could explain the relationship between the two known variables. However, the student made an error in performing the arithmetic operation. The correct approach would have been to add the difference in their heights, which is 35 cm, to 198 cm. Instead, the student subtracted 35 cm from 198 cm, leading to an incorrect result.

On their answer sheet, the students correctly noted the combined heights of Zeeana and Fahrezan, as stated in the problem. However, the student struggled with performing the transformation operation correctly, leading to errors in subsequent steps. According to the survey, most students find it challenging to create mathematical models for problems, which contributes to their perception of mathematics as a difficult subject.

From the perspective of their productive struggle, students who made errors at the transformation stage had an average score of 74, which places them in the medium category. In terms of asking questions, these students began to show a willingness to inquire from their teachers or peers when confused. Regarding motivation, these students still possessed the desire to solve problems and were willing to try again when facing difficulties. In terms of strategy, they would check their answers, although they sometimes tended to be less thorough.

Of the three problems provided, almost all were completed up to the stage of noting known and asked details, and the students were able to determine problem-solving strategies. However, they sometimes still required intervention and motivation to enhance their productive struggle and needed reminders to be more careful in their calculations (Sherman, H. J., Richardson, L. I., & Yard, G. J, 2019); Winterer, E, 2024).

Analysis of Errors in Students' Answers with High Productive Struggle at the Process Skills Stage

43.75% of students, or 14 students, made errors at the process skills stage. Below is an example of a student's response who made an error at this stage, thus receiving a score of two.

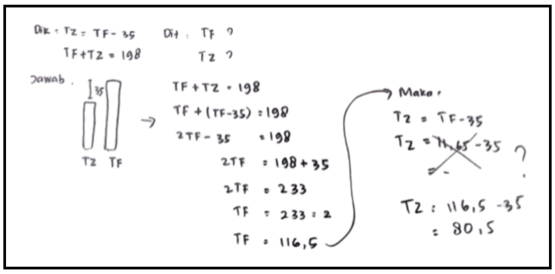


Figure 3. Incorrect Answer from a Student at the Process Skills Stage

In Figure 3, it is evident that the student understood the problem and had no difficulty comprehending the statement that Zeeana's height is 35 cm shorter than Fahrezan's, enabling them to create an algebraic equation as a mathematical model. The student could explain the relationship between the two known variables and accurately determined the first variable, Fahrezan's height. However, the student made an error during the arithmetic operation. Zeeana's height should have been calculated by subtracting 35 cm from 116.5 (Fahrezan's height).

There was a transcription error where the student wrote 11.65 instead of 116.5 and subtracted 35, which confused solving the arithmetic operation. Subsequently, the student attempted to recalculate by subtracting 35 cm from 116.5, but the result was incorrect; it should have been 81.5 cm.

On their answer sheet, the student had correctly noted the subtraction of 35 cm from Fahrezan's height but was careless in calculating the final result. According to the survey, most students have difficulty relating one mathematical concept to another, which leads to errors when they are not meticulous.

From the perspective of their productive struggle, students who made errors at this stage had an average score of 85, placing them in the high category. These students consistently ask questions of their teachers or peers when confused. In terms of motivation, they possess a strong will to solve problems and a high desire to try again when facing difficulties. In terms of strategy, they check their answers, though they sometimes still tend to be less meticulous.

Of the three problems provided, almost all were completed up to the stage of noting known and asked details, and the students were able to determine problem-solving strategies. These students also need support to maintain their productive struggle and reminders to be more meticulous in their calculations (Sayster, A, 2023)

Analysis of Errors in Students' Answers with High Productive Struggle at the Conclusion Stage

90.62% of students, or 29 students, made errors at the conclusion stage. Below is an example of a student's response who made an error at this stage, thus receiving a score of three.

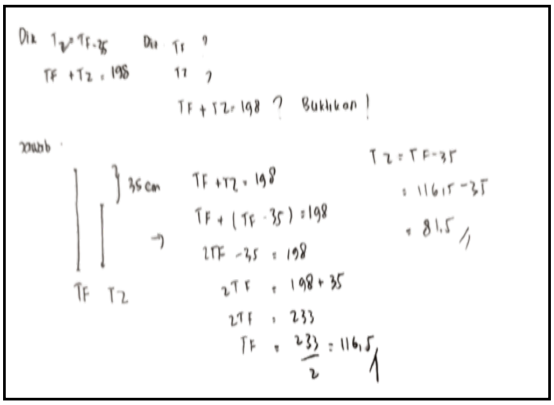


Figure 4. Incorrect Answer from a Student at the Conclusion Stage

In Figure 4, it is evident that the student understood the problem and had no difficulty comprehending that Zeeana's height is 35 cm shorter than Fahrezan's, allowing them to create an algebraic equation model. The student could explain the relationship between the two known variables and accurately determine the values of both variables. However, the student made an error by not verifying the sum of the two variables.

According to interviews, the student did not have difficulty in calculating the values of each variable through addition and subtraction operations. The students also had a good understanding of concepts, enabling them to solve the problem easily. According to the survey, the student enjoys learning mathematics because it helps solve daily life problems related to math, thus they are keen on learning mathematics and have mastered the mathematical concepts involved in the problem.

From the perspective of their productive struggle, the group of students who made errors at the understanding stage had an average score of 85, placing them in the high category. In terms of asking questions, these students have begun to answer their queries. Regarding motivation, they have a very high willingness to solve problems and a strong desire to try again when facing difficulties. In terms of strategy, these students check their answers and tend to be very meticulous. Of the three problems provided, almost all were completed up to the final stage, which is drawing conclusions. These students also excel in maintaining their productive struggle.

Based on the analysis of the errors in answers, most students who struggled to solve the problems did so because they did not understand the material related to addition and subtraction operations within the algebraic problems. Students were also not meticulous in solving problems because they rushed to submit their answers without reviewing them. Students dislike mathematics lessons because they perceive the subject as difficult to understand. Furthermore, the varying levels of productive struggle among students also affect their mathematics learning outcomes.

Conclusion

Based on the research conducted on class V students at an elementary school in Bandung, it can be concluded that their mathematical problem-solving abilities are still low. Of the 32 students, only three were able to effectively solve problems through all stages of the mathematical problem-solving process, from understanding and transformation to process skills and conclusion. Students who face challenges in the understanding stage of mathematical problems tend to have low productive struggle. These students are not accustomed to working on problem-solving tasks, making it difficult for them to comprehend the information in the questions. Students need to be trained on tasks that require high-level thinking so that their mathematical problem-solving abilities can develop effectively. Meanwhile, students who face challenges in the transformation stage tend to have medium productive struggles. Unlike those, students encountering difficulties in the process skills and conclusion stages tend to exhibit high productive struggle.

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