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The Hypothetical Learning Trajectories of AI Usage in Learning Integral for Aerospace Engineering Students

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Abstract

The increasing use of AI among students has transformed learning habits, often shifting from deep conceptual understanding to quick solution retrieval. Mathematics education in aerospace engineering requires innovative approaches to enhance students' conceptual understanding and problem-solving skills. This study implemented Realistic Mathematics Education (RME) for Aerospace Engineering Students (AES) using a design research methodology. It is focused only on the development of the hypothetical learning trajectory (HLT) in learning integration strategies for the first and second year of AES using artificial intelligence (AI). After working with the HLT during the first and second cycle, this study discovered that the students' high expectations of AI while solving integration approaches did not match. Students still require more assistance to grasp the AI answer, such as lecturer clarification or video explanation on YouTube. Students frequently use AI to solve problems without fully comprehending the actual procedure. Due to the time constraints, they use the AI answer immediately rather than paraphrasing it to their understanding. Consequently, we found that students realise their inability to depend completely on AI for deep understanding. As a result, AI is used to facilitate the recollection of existing knowledge or the confirmation of the final response rather than to understand new material. AI supports teaching but is not a substitute.

Keywords: Aerospace Engineering, AI, HLT, Integral, Mathematics Education.

Introduction

Mathematics is the queen of sciences (Gauss, 1856). It is due to that mathematics was successful at uncovering the nature of physical reality and also provides a foundation for other sciences. Engineering is the application of sciences that are strongly related to mathematics. Therefore, mathematics in aerospace engineering is ubiquitous. Aerospace Engineering Students (AES) must master the mathematics for special purposes, which is Mathematics for Aerospace.

A scoping analysis of Indonesian mathematics education research from 2015 to 2021 indicated a primary emphasis on junior high school students (35.63%), while college students or pre-service teachers constituted 23.87% of the participants (Nur et al., 2021). However, the current research is still mainly focused on general mathematics education subjects, with limited attention to specialised applications within engineering fields. The sequence of mathematical concepts must be learned to help students deepen their comprehension (Khairudin et al., 2020). Therefore, mathematics for aerospace engineering needs a learning sequence towards their curricula regarding mathematics.

Developing a local mathematical instructional theory for AES is essential to ensure that mathematical concepts are taught in direct relation to aerospace applications, enhancing students' ability to apply theoretical knowledge to practical engineering problems. By contextualising mathematics education within the aerospace field, students' motivation and engagement levels increase, leading to better educational outcomes (Armiati & Sari, 2022; Hidayati et al., 2022; Nuraida & Amam, 2019; Shanty et al., 2011; Sinaga, 2024). To achieve this, we begin with a thought experiment, designing a Hypothetical Learning Trajectory (HLT) with the framework of Realistic Mathematics Education (RME). The HLT is for AES when solving mathematics problems using Artificial Intelligence (AI), that outlines the path students are expected to follow, which will then be incorporated into classroom activities.

Nowadays, students are aware of the presence of AI, which has also grown quickly in recent years (Baidoo-Anu & Ansah, 2023; Graepel, 2024; Moghe et al., 2023; Páez et al., 2022; Wardat et al., 2023; Woolf, 1990). Also, there is a growing interest in using AI as an educational tool and method (Subroto et al., 2024). AI applications have been used in schools to improve both administrative and academic support (Lo, 2023). For example, AI is used in intelligent tutoring systems (ITS), which follow one-on-one personal tutoring (Lo, 2023). Students desire AI to provide systematic explanations, feedback, assistance, or another mathematical formula that can be used as an alternative to answering questions (Kaledio et al., 2024; Lo et al., 2024). Research on AI in mathematics education on mathematical literacy, assessment, and gamification is still underexplored (Subroto et al., 2024). Many students find that while AI can solve integrals and other problems, it does not always explain the reasoning clearly, making it difficult for them to understand the steps or learn from the process (Schorcht et al., 2024).

Based on the issues raised, to create a learning sequence for mathematical material for AES using AI as an educational tool, we must establish and test it. HLT is one technique to do so, as it includes conjectures about the process of learning how to learn as well as tactics that students develop and thrive while learning activities are conducted in class. As a result, this study wants to create HLT for AES using the concept of integration techniques with AI usage, which will then be examined to confirm the conjecture. The aim is to examine AI usage to optimise the learning process and drive a deeper conceptual understanding of integration techniques for AES.

Methods

Following the design research framework (Plomp & Nieveen, 2013; van den Akker et al., 1999), this study developed an HLT for AES mastering integration techniques combined with the context of engineering education known as conceive, design, implement, and operate (CDIO) (Crawley et al., 2008). In Plomp's (Plomp & Nieveen, 2013) Design Research framework, the HLT is primarily developed and refined in the Prototyping Phase, after Preliminary Research and before Assessment Phase. During the Prototyping Phase, the initial HLT is designed, iteratively tested through classroom cycles, and refined based on student responses to improve its effectiveness.

The main component of HLT is the conjecture of students' process of understanding that has been compared with the real activities. This comparison will create several conjectures about students' response and understanding that has high validity (Hendrik et al., 2020; Prahmana & Kusumah, 2016; Syafriandi et al., 2020). This study follows the five characteristics of Design

Research (Funny, 2021) or Educational Design Research, which are interventionist, iterative, process-oriented, utility-oriented and theory-oriented. Allowing students to use AI is interfered with purposively to increase students' comprehension of integration techniques. There were two cycles in this study to revise the HLT as an iterative cycle. The revised HLT is based on the students' process, and the concept of integration techniques is a fundamental theory for mastering the advanced aerospace engineering course.

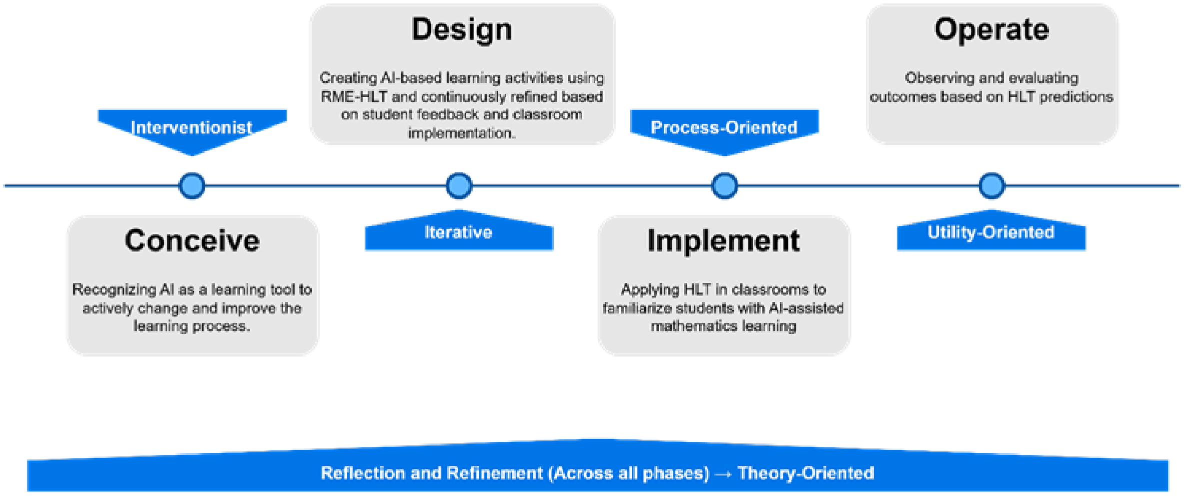

Figure 1. Collaboration CDIO and Five Characteristics of DR

By collaborating on the purpose of design research and the component of engineering education, we have restructured the understanding of the CDIO context as follows in the figure 1 above:

1. Conceive that AI becomes tools for learning (admit the AI's existence for the learning process).
2. Design a learning activity that involves AI and use the AI as learning material (RME – HLT).
3. Implement the HLT for students while they are learning mathematics ideas so that they are familiar with them when they need to recall them later.
4. Operate the design and see what happens based on the HLT's guess.

The total number of subjects involved in this study was 93 students. They were the first-year AES on a private campus in Yogyakarta. They have learnt the simplest integral in their senior high school which was the power rule of integral, but they had not known other strategies of integral. The data was collected from the classroom activities in a video recorded about the students' learning process and the students' answers to the given problem on the HLT. Students were given learning activities that asked them to compare the answer to an example problem in the Calculus book by Purcell and the solution from AI. They were free to use any AI they want. The HLT conjectured students' responses and understanding based on the learning activities. The conjecture in the HLT will be confirmed within the two cycles. The first cycle with a small group (five of AES) as a pilot classroom result was used to revise the HLT. The revised HLT was implemented in the second cycle with all AES. The conjectures and actual learning processes derived from the study of the two cycles were reported in the discussion of the results.

Later, the data collected from the two cycles was analysed qualitatively with the RME framework to produce a valid HLT. RME requires a context to create the learning sequence that will support the HLT. The context in this study was the AI itself as in real life, pupils are already familiar with using AI to solve arithmetic problems.

Results and Discussion

Cycle 1, as a pilot study, aimed to collect actual responses from the small group (five AES) as a reference to revise the HLT before full-scale implementation for the whole class in cycle 2. There were five AES involved in this cycle with heterogeneous mathematics skills. Students were asked to compare two answers: one from the example in Purcell's calculus book and another generated using AI, with the freedom to choose any AI tool. Cycle 1 revealed that most students still require assistance from other sources than the AI-provided solution. Thus, students were required to describe the AI solution they employed on the example problems on Calculus Purcell in front of their friends. Unfortunately, they were unable to execute it directly that day. As a result, they were offered the opportunity to videotape it. After submitting the videotape, the students were asked if they understood what they were saying in the video; four out of five AES stated they did not. The video shows that they just read the sentences without understanding. The remaining participants were asked if they could grasp it just through AI explanations. Their answer was no; they needed more information to understand the AI solution. They watch YouTube videos or visit instructional websites to have a deeper understanding of the AI solution.

Based on the results of cycle 1, the HLT was modified. This attempts to assess the AI explanation's impact on the students' understanding. In cycle 2, the students were asked to explain directly (not by videotaping) the solution chosen in front of the class. The difference in cycle 2 is that students present in groups instead of individually. Table 1 illustrates the modified HLT for cycle 2 and the result of the actual response by AES.

Table 1

The Revised HLT of the AI Use in Learning Integration Technique on Cycle 2 and the Result

No	Activities	Conjecture	Actual Response from Cycle 2
1	Students are given the solution to an example problem in chapter 9.3 positive series integral test from the book "Calculus Purcell" as well as the answer solved by AI.	Students tend to view the shortest explanation, the better.	60% of students think that explanations from books are more effective than AI.
2	Students are required to understand both methods of explanation: • Explanation from the book • Explanation from AI	Students will have no difficulty understanding the step-by-step explanation they have chosen.	• When being asked to clarify, both students who chose AI and book struggled to comprehend the explanation. • They did not understand step by step of the explanation.

3	Students have to compare both explanation and find out which one is better, then have to explain on what aspect it is better than others. They were given some trigger questions such as: • Why is it so? • Where does this come from? • How is the strategy? Is it correct?	Students must understand the procedure they chose when comparing.	• They struggled to answer trigger questions and used their smartphone, but still couldn't find a solution. • They only caught a glimpse of the basic explanation and did not understand it.
4	Students are being re-explained by the lecturer about the integration techniques.	Students will recall their prior knowledge about integration techniques.	• Students were more enjoyable to re-learn the integration techniques with the lecturer. • Students used AI to recall their prior knowledge on several simple integral concepts based on lecturer questions, such as: – What is the integral of x^5 ? – What is the formula of partial integration? – What is the integral of ln x?
5	Students are given an integral problem individually. Students are asked to solve it by using AI, and then figure it out to be able to rewrite it in their own way.	Students will be able to rewrite the solution, or at least summarize it based on the AI answer.	• Half of the students' rewrote solutions differed from AI's. • Students were unaware that their solution was erroneous and that the correct one was from AI. • The students' reaction was erroneous because the AI answer was incomplete, thus they completed it themselves.

Several discoveries emerged from the students' responses to step five in the revised HLT of cycle 2, which challenged them to solve an essential problem using AI and their methods. These findings demonstrated the complexity of AI in the students' comprehension. Figure 2 shows that AI helped students retain the subject completely, yet students still made conceptual mistakes, particularly in symbol interpretation and partial fraction decomposition. Their handwritten work misuses "x" as both a variable and an operator, skips important justifications, and makes mistakes in rewriting fractions before integration. This suggests they were following AI-generated steps without truly understanding them, highlighting the need for more guidance to build a stronger conceptual foundation.

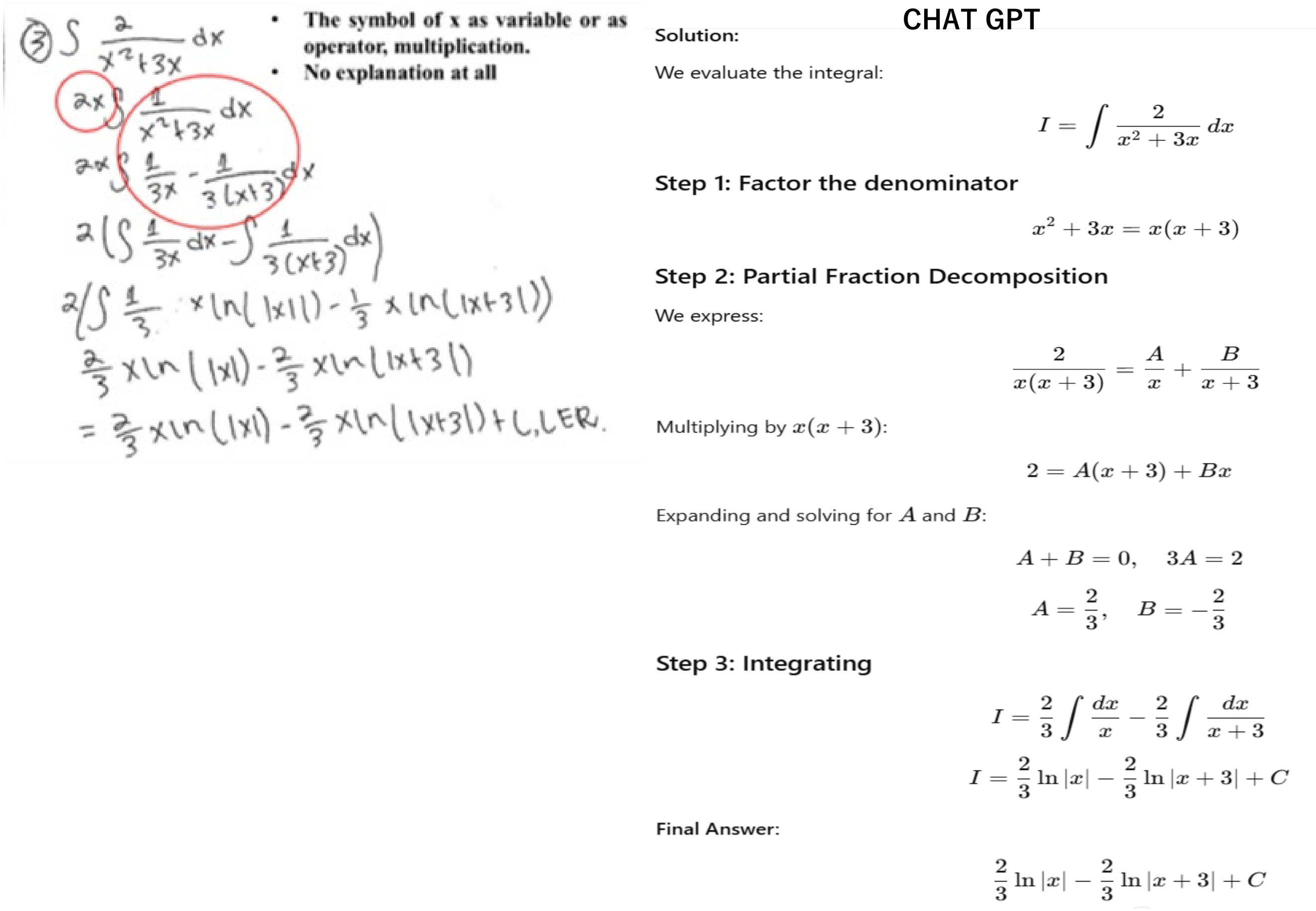

Figure 2. Even with AI support, students still have incorrect concepts.

Meanwhile, in Figure 3, students assessed the integral using a different technique than the AI. From the Figure 3, the AI directly applied substitution, setting

$$u = 3e^x + 1 \tag{1}$$

which simplifies the integral. It then converted the integral into a power function of u, leading to a straightforward antiderivative calculation. Finally, the AI substituted back

$$u = 3e^x + 1 \tag{1}$$

to express the solution in terms of x. The students use the difference substitution, he set

$$u = \sqrt{3e^x + 1} \tag{2}$$

and got its derivative

$$\frac{du}{dx} = \frac{3e^x}{2\sqrt{3e^x+1}} \tag{3}$$

or

$$\frac{2u}{3e^x}du = dx \tag{4}$$

that he used to substitute the dx. Therefore, he got the simplifies integral become

$$\frac{2}{3}\int u^2\,du \tag{5}$$

while the AI got

$$\frac{1}{3}\int \sqrt{u}\,du \tag{6}$$

though the ultimate answer remained equivalent to the AI's solution. The student's approach was more detailed, ensuring they fully understood how the substitution transformed the integral. The students carefully worked through each step, explicitly differentiating and isolating terms before integrating. While it took a bit longer, this method helped reinforce their grasp of the process rather than just following a formula.

1) $\int e^x \sqrt{3e^x + 1}\, d$

$\int e^x \sqrt{3e^x + 1}\, dx = \frac{1}{3} \int \sqrt{u}\, du$

$\int \sqrt{u}\, du = \frac{2}{3} u^{3/2}$

$\frac{1}{3} \times \frac{2}{3} (3e^x + 1)^{3/2} = \frac{2}{9} (3e^x + 1)^{3/2}$ => menggunakan AI (ChatGPT)

Translate: Using AI (ChatGPT)

$\int e^x \sqrt{3e^x + 1}\, dx = \frac{2}{9} (3e^x + 1)^{3/2} + C$

Jadi, hasilnya adalah $\frac{2}{9} (3e^x + 1)^{3/2} + C$

2) $\int e^x \sqrt{3e^x + 1}\, d$

$\int e^x \sqrt{3e^x + 1}\, dx = \int e^x u \cdot \frac{2u}{3e^x}\, du = \frac{2}{3} \int u^2\, du$

$\frac{2}{3} \int u^2\, du = \frac{2}{9} u$ => Pengerjaan mandiri

$\frac{2}{9} u^3 = \frac{2}{9} (3e^x + 1)^{\frac{3}{2}}$

Translate: Students' method

$\int e^x \sqrt{3e^x + 1}\, dx = \frac{2}{9} (3e^x + 1)^{\frac{3}{2}} + C$

Jadi, hasilnya adalah $\frac{2}{9} (3e^x + 1)^{\frac{3}{2}} + C$

Figure 3. With AI support, pupils can apply various methods.

Figure 4 indicates that the AI clearly stated the solution, but the students were neither overconfident in their response nor unaware that they were using the wrong concept of partial fraction decomposition. AI systems frequently provide answers without clear, understandable reasoning (Al-Zahrani, 2024). This lack of clarity might make it difficult for students to understand the fundamental concepts, resulting in misunderstanding and mistrust. When students rely extensively on AI without critically interacting, they risk superficial understanding and diminished problem-solving skills, as well as missing cognitive processes required for comprehension (Kim et al., 2024; Zhai et al., 2024).

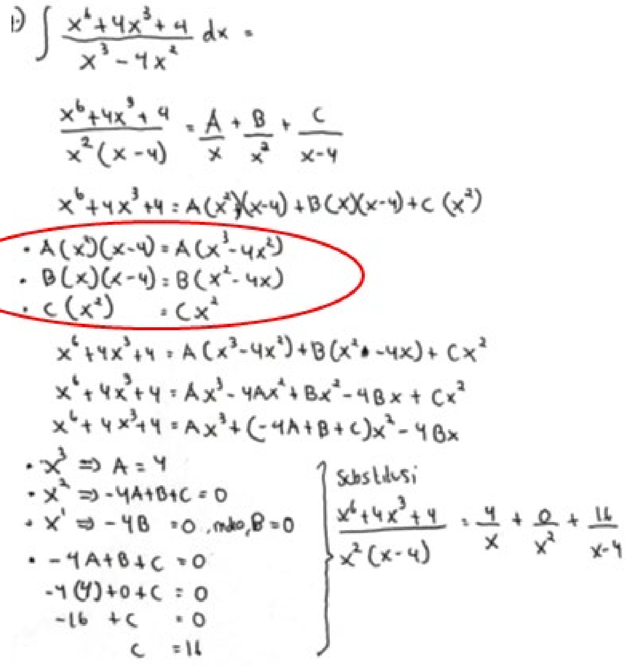

Figure 4. Due to poor AI support, pupils utilize their methods but continue to make mistakes.

According to this study's findings, more than half of the students believed AI explanations were as good as book or teacher explanations. Students' expectations of using AI did not match reality.

Teacher : So, how was it using AI for calculus? Did it help?
Student 1 : At first, I thought AI was always right, but sometimes it gives wrong answers.
Teacher : Really? Like what?
Student 2 : AI made a mistake in an integral problem. I asked it about an integral problem, but its explanation confused me—it was neither too long nor too short, complicated.
Teacher : So, did AI help or make things harder?
Student 3 : Honestly, I just copied the answer without understanding it.
Student 1 : Same! We relied too much on AI instead of figuring it out ourselves.

They expected AI to offer the correct response instantly, but AI could occasionally provide an incorrect answer. They initially thought an AI explanation could help them, but they were mostly unable to comprehend it. AI explanations can be too complex, too long, or too short, leaving students with questions. As a result, students relied exclusively on AI to know the answer or how to respond, without understanding. Furthermore, students perceived that AI, as a tool, is sometimes less effective and more confusing.

However, based on talks with students, the application of AI in this study has both advantages and disadvantages. The advantages were the ability to access the solution fast, provide a thorough explanation, and use it at any time and from any location as self-learning. Meanwhile, the disadvantages include AI charging a charge for additional capabilities, students having difficulties entering mathematical notation, AI output being incorrect, and unnecessarily long explanations with little notation that needed effort to grasp. Most people become passive users of AI following the creative engagement period of education, and they use it without understanding how it works. To overcome the passive consumer, teacher can write questions to AI about the original issue or in foreign language in a mathematical calculation.

Therefore, to enhance students' understanding of learning a mathematical concept using AI, these conditions must suffice:

1. Students themselves have to input the mathematics problems into AI.
   They must decide how to best question the AI, whether by inputting the mathematics notation, taking a photo, or uploading the file. The typing methods will require students to be equation-friendly by utilising the Phyton language, since AI will evolve and become more natural to everyone familiar with standard mathematical notations and vocabulary (Hanna et al., 2023). To ensure that their students can utilise AI correctly, lecturers must be trained on how to detect the usage of AI in assignments and how to completely use AI in their teaching preparation and course assessment (Lo, 2023). Provide pupils with the abilities required for success in a quickly expanding digital world (Opesemowo, 2024).
2. Attend or accompany your students when learning the AI.
   Verify their comprehension by asking the students about the explanation provided by AI. For example: is the explanation understandable?; why does AI behave this way? what is happening?; do you require clarification from the teacher or a YouTube video?. Those questions attempt to ensure that they comprehend the AI explanation. It will also encourage students to be responsible for their understanding of AI rather than simply using the answer. Additionally, to prevent AI "hallucinations," where AI produces misleading or incorrect solutions that appear plausible but may confuse learners, students should be encouraged to personalize their learning experience (Zhai et al., 2024) . This can be achieved by encouraging an interactive learning process where students ask AI follow-up questions to gain a deeper understanding, instead of just accepting its first answer (Stefanova & Georgiev, 2024). Students have been aware of AI's imperfections, such as its reliance on biased data, limited up-to-date knowledge, and the possibility of producing wrong or false information. Thus, teachers should encourage pupils to use other authoritative sources (such as books) to check, analyse, and validate the factual accuracy of information provided by AI (Lo, 2023).
3. Ask the students to rewrite the AI answer with their answers.
   This process is meant to assist students in reinforcing their understanding of AI concepts, raising awareness of ethical issues, and validating algorithmic bias. Furthermore, asking students to rewrite their answers can boost creativity and lead to exploring new problem-solving approaches.
4. Make sure the students can replicate the way AI answers by giving them similar problems.
   Ensure pupils solve the problem without the help of AI to minimise over-reliance on AI and increase students' confidence in their abilities to solve problems independently.

Conclusion

The study found that while AI is widely used by AES to solve integral problems, it does not significantly enhance their deep conceptual understanding. Students often rely on AI for quick answers rather than engaging with the underlying mathematical principles. Many struggled to comprehend AI-generated solutions without additional support from lecturers, classmates' explanations, textbooks, or online resources. Using AI without a teacher or lecturer will not effectively assist students in learning new concepts in the classroom. The modified HLT in cycle 2, which compared AI solutions with textbook explanations, revealed that students still required further clarification. Students recognised that AI serves better as a tool for recalling existing knowledge or validating answers rather than learning new concepts. Finally, AI is still best used as a facilitator (to assist students in getting the correct answer or recalling past knowledge), a validator (to and customise the learning experience), and an additional tool (to encourage students to participate in learning activities).

However, the long-term effects of AI usage on the learning process remain unknown, as it is reportedly intended to train students to think quickly. Limitations of this study include: 1) a narrow sample size, 2) AI inconsistencies, and 3) the absence of long-term impact assessment, which affects the generalizability of the findings. Since the research focuses only on AES, its applicability to other disciplines remains uncertain. Additionally, variations in AI-generated responses may influence learning outcomes differently across contexts. Without long-term evaluation, it is unclear whether AI enhances deep mathematical understanding over time. Addressing these limitations in future research will help refine AI-assisted learning strategies for broader and more effective applications.

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