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Analysis Prospective Mathematics Teachers' Lesson Planning: A Praxeological-Didactical Theory

[1]Eva Musyrifah, [2]Didi Suryadi, [3]Endang Cahya, & [4]Jarnawi Afghani Dahlan
[1,2,3,4]*Mathematics Education Department, Universitas Pendidikan Indonesia, Bandung, Indonesia*
[1]*Mathematics Education Department, Universitas Islam Negeri Syarif Hidayatullah, Jakarta, Indonesia*
[2]*didisuryadi@upi.edu*

Abstract

A high quality of lesson planning is the key to achieving effective learning. One of the characteristics of effective learning is that it minimizes learning obstacles. This study aims to explore an Indonesian prospective mathematics teacher's lesson planning and identify its learning obstacles. Data was analyzed from the lesson plan document of prospective mathematics teacher students carrying out teaching practices at one of the high schools in Jakarta. Analysis was done using the praxeological-didactical (PDA) method. PDA offers space to analyze human actions, such as lesson plans. Four types of tasks are presented in the lesson plan on drawing graphs of quadratic functions. The predicted learning obstacles identified from the lesson plan are (1) epistemological obstacles, caused by the limitations of certain contexts; (2) ontogenic obstacles, caused by the presentation of tasks due to inappropriate prerequisites; and (3) didactic obstacles, caused by tasks that are not relevant to the next task. The implication is that the results of the study can be used to improve a quality lesson plan, and the use of a PDA can be a consideration for prospective mathematics teachers when designing a quality lesson plan.

Keywords: Lesson Plan, Mathematical Praxeology, Learning obstacle, Prospective Mathematics Teacher

Introduction

Lesson planning is an essential aspect of improving the effectiveness of mathematics teaching and learning (Akyuz et al., 2013; Kang, 2017; König et al., 2021). It can influence the quality of students' learning experience (González et al., 2020; Xu & Clarke, 2019), including supporting the creation of situations for students to learn actively and creating an attractive learning environment (Fouryza et al., 2019; Iqbal et al., 2021). In addition, lesson planning helps teachers to implement their ideas on classroom learning, and increase their confidence, and lesson planning can also be used as a document to improve their learning in the future (Rasuli, 2020). This is because lesson planning contains evaluation elements to understand teaching effectiveness. Teachers can evaluate student progress and reflect on their teaching practices (Mulyani, 2019). Thus, lesson planning is a fundamental aspect that serves to improve the quality of teaching and student learning outcomes. Therefore, teachers need to master lesson planning skills to achieve optimal goals.

Given the important role of lesson plans, prospective teachers need to learn how to create effective lesson plans (Morris & Hiebert, 2017; Rusznyak & Walton, 2011). Research by Aliustaoğlu & Tuna (2022) shows that lesson plans have improved prospective teachers' understanding of mathematics teaching. In addition, lesson plans assist prospective teachers in bridging the gap between theoretical knowledge and practical application (Sahin-taskin, 2017).

By planning lessons properly, prospective teachers can study the material in advance, determine effective teaching strategies anticipate student responses, and minimize errors in the classroom.

Several studies discussed how prospective teachers design lesson plans and analyzed their challenges in planning lessons. Kilicoglu (2019) points out that prospective teachers' planning and teaching skills are still inadequate and inconsistent. Furthermore, Emre-Akdoğan & Yazgan-Sağ (2018) highlighted prospective teachers' challenges in designing lesson plans, namely designed and emphasized using technological media to keep students motivated when learning. In addition, Sahin-Taskin (2017) identified that prospective mathematics teachers have difficulties in determining activities that are appropriate to students' ability levels because prospective teachers have insufficient time to understand their students and lack experience, so they have difficulty understanding students and identifying needs that match their characteristics, as well as difficulty in planning lessons effectively. Meanwhile, one of the findings from the results of research conducted by (Sumarni et al. 2019) was that there are prospective teachers who lack understanding of the mathematics material they design. Therefore, analyzing and addressing lesson plan design in prospective teachers is still important and interesting to discuss.

Other research by (Durand-Guerrier et al., 2010; Rasmussen, 2016; Shinno & Mizoguchi, 2021) explored lesson planning by prospective and in-service mathematics teachers. They analyzed the thought processes involved, both practical and theoretical that utilized the theory of mathematical praxeology. According to Chevallard's Didactic Anthropology Theory (Chevallard, 2019; García et al., 2006; Bosch & Gascon, 2006), understanding the praxeology of mathematics is essential in educational contexts, including in helping prospective teachers in lesson planning (Suryadi et al., 2023). Praxeology is part of the Didactics Anthropology Theory, which states that every human activity can be described in praxeology, which consists of two blocks, namely the praxis block and the logos block (Bosch et al., 2017; Chevallard, 2019). The praxis block includes a type of task and technique, while the logos block includes technology and theory. In particular, mathematics lesson plans contain student activities in learning that involve the thinking process, so praxeology can be used to study them. In addition, studying the exploration of thinking processes related to practical and theoretical aspects is very helpful for prospective teachers in developing lesson plans.

Lesson plans are designed by teachers before implementing learning (Farhang et al., 2023). The didactic design by the teacher is one of the external factors that can cause learning obstacles (Suryadi, 2019). Thus, teacher awareness of learning obstacles is critical in reflective practice to develop effective didactic designs (Marfuah et al., 2023) because the didactic design aims to improve the previous design. Identifying learning obstacles is important to be carried out as a basis for preparing teaching materials, predicting student responses to the learning that will be carried out, and anticipating it so that the lesson plan becomes effective (Rohimah et al., 2022). Thus, in designing lesson plans, teachers are not only able to master the material to be taught but also able to predict various possible student responses based on learning obstacles and how to anticipate them (Musyrifah, Dwirahayu, et al., 2022). Based on the description above, it is also important for prospective teachers to consider learning obstacles in designing lessons.

Previous research has discussed how prospective teacher students design lesson plans and in-depth exploration of lesson plans developed by prospective teachers using praxeology analysis. However, some studies have not addressed the prediction of learning obstacles in the designed lesson plan. In addition, using praxeology theory and prediction of learning obstacles aimed at improving the quality of students' textbooks has been carried out by (HastiYunianta et al., 2023). Thus, this study aims to explore the lesson planning of prospective mathematics teachers through the perspective of praxeology in didactic anthropology theory equipped with predictions of learning obstacles that may arise from the lesson plans developed by students. Therefore, the research questions to be answered in this study are: (1) how is the mathematical praxeology of the lesson plans designed by prospective teachers, and (2) how is the identification of learning barriers that may occur to students on these lesson plans?

Theoretical Framework

Praxeology is part of the Didactic Anthropology Theory (ATD), a theoretical framework for studying human actions. Chevallard introduced this theory in the early 1980s. Since then, many researchers in mathematics education have used it to study various human actions, as proposed by Chevallard. A lesson plan or teaching material can be categorized into human actions that can be studied using praxeology. Several previous studies have used praxeology to analyze the presentation in textbooks, such as research conducted by (Bosch et al., 2021; González-Martín et al., 2013; Wijayanti & Winslow, 2017).

Praxeology is a combination of two words, namely praxis and logos. The praxis component refers to the practical part of the activity or knowledge, while the logos or knowledge component refers to the reasoning that explains the techniques used (García et al., 2006). Praxis involves how a person reasons and thinks and logos guides the thought process (Huang et al., 2021) The praxeological component is denoted in the 4T-four-tuple model $(T, \tau, \theta, \Theta)$ in which the four letters have related relationships (Barbé et al., 2005; Chevallard et al., 2015; Wijayanti & Winslow, 2017) Thus, the praxis component consists of a task type (T) and a technique (τ). $T$is a task or problem that must be solved (Pansell & Boistrup, 2018) τ is a method used to solve T(Putra, 2019) Wijayanti and Winslow (2017) noted that the term "task" in ATD means something that humans can accomplish with simple actions (techniques), in mathematics, it can be some algorithm or other basic method. Furthermore, the components of logos consist of technology (θ) and theory (Θ). θ refers to the reasoning used to explain τ that students use to solve it. Θ including mathematical concepts built around learning objectives that justify, explain, and support a variety of technology (θ), such as definitions and axioms (Østergaard, 2013). The explanation of praxis and logos in this study is presented in the following table.

Table 1.

Plaxeology Component (adapted from (Østergaard, 2013; Pansell & Boistrup, 2018)*)*

Components of Praxeology		Explanation
Praxis	Task	Task or problem to be solved
	Technique	Methods used to complete the task
Logos	Technology	Mathematical reasoning of technique selection
	Theory	Definition, theorem, formula that is the target or part of the target knowledge to be achieved

Learning situations are formed because of interactions involving all didactic components (Marfuah et al., 2022). Brousseau (2002) describes the situation as a series of actions, formulation, validation, and institutionalization. In action, students are presented with a problem that allows them to use the learning experience they have gone through to determine a strategy or solve a problem. Formulation is a situation where students discuss by uniting opinions from the action process so that the best solution or strategy is obtained. Validation facilitates students to present their opinions on the resulting strategy or solution in front of the class so that the teacher validates it. Finally, institutionalization allows students to develop the knowledge they already have on new problems with different contexts.

However, there are times when didactic situations that are designed cause learning obstacles in students. Learning obstacles are categorized into three types, namely epistemological, didactic, and ontogenic (Brousseau, 2002) Epistemological Obstacles arise when students make mistakes on assignments or questions that are typical or unfamiliar to them. These obstacles are often caused by students' limited understanding of a concept, students tend to follow what has been exemplified. According to didactic situation theory, the situation of the Obstacle, which is known as the limitation of context (Musyrifah, et al., 2022; Suryadi, 2019) In the lesson plan designed by prospective teachers, this obstacle can be detected from a series of didactic situations, namely if the design facilitates students to go through action situations, formulation, validation, and institutionalization in producing knowledge. Didactical Obstacles occur due to the sequence or stage of presentation of material that results in the continuity of the student's thinking process (Suryadi, 2019). In lesson planning, this obstacle can be identified by examining the alignment between the task and the development of the thought process, as well as whether the presentation of the material is well structured. The researcher analyzed the order of presentation on each task, and whether there was a relationship between unrelated tasks compared to the expected target ability. Meanwhile, ontogenical obstacles are related to students' mental readiness and cognitive maturity in acquiring new knowledge (Suryadi, 2019). This obstacle may stem from students' interest in the material, mistakes during the learning process, or previous learning experiences. In identifying this type of obstacle, the researcher uses two ways, namely analyzing student involvement in each task and analyzing the prerequisite concepts that support the formulation of target knowledge.

Methods

This qualitative research used a hermeneutics phenomenology approach (Hendriyanto et al., 2023) to analyze lesson plans designed by prospective mathematics teachers. Qualitative research explores and understands the meaning individuals or groups ascribe to a social or human problem, and the researcher interprets the meaning of data (Creswell & Plano Clark, 2018). In hermeneutic phenomenology, the researcher describes and interprets the whole process of phenomena (R. Dangal & Joshi, 2020). The phenomenon may originate from practical world affairs, a theoretical discipline, a personal experience or insight, an unsatisfying circumstance, a promising opportunity, a breakdown in expected arrangements, or simply a topic of interest (Van de Ven, 2016). The hermeneutic phenomenology of research is conducted through empirical (collection of experiences) and reflective (analysis of their meanings) activities (Fuster Guillen, 2019; Sloan & Bowe, 2014). Specifically, this research employed a document hermeneutics phenomenological approach, namely a lesson plan, which includes an in-depth systematic analysis of lesson plans, interpreting them to achieve understanding, and developing empirical knowledge of the phenomena studied.

The data was analyzed using the Praxeology approach from Chevallard's Anthropological Theory of Didactics (ATD) and Brossoeou's Theory of Learning Barriers in Didactic Situations (HastiYunianta et al., 2023). Praxeology provides a tool to analyze the data (Tesfamicael & Lundeby, 2019), i.e., analyze mathematical tasks deeply, predict the techniques students use, determine the reasons for choosing these techniques, and determine the theoretical basis for constructing mathematical concepts or procedures. Thus, through this approach, we first traced the set of tasks given in the lesson plans, explored their relationships, identified the knowledge needed by students, and how the set of tasks facilitated students to construct their knowledge. Next, we justify the predicted techniques that students use to complete the tasks, explore the reasons why students choose these techniques, and identify the theoretical basis that supports the justification of the technology. From our analysis, we obtained a series of learning trajectories that were formed, the relationship between the tasks presented, the accuracy of the theory used in building students' knowledge, and students' involvement in constructing knowledge.

Next, we evaluated all the mathematical practices involved. We used Brousseou's Theory of Didactic Situations and learning barriers to identify possible learning obstacles. The sequence of activities in this study is presented in the *Figure* 1.

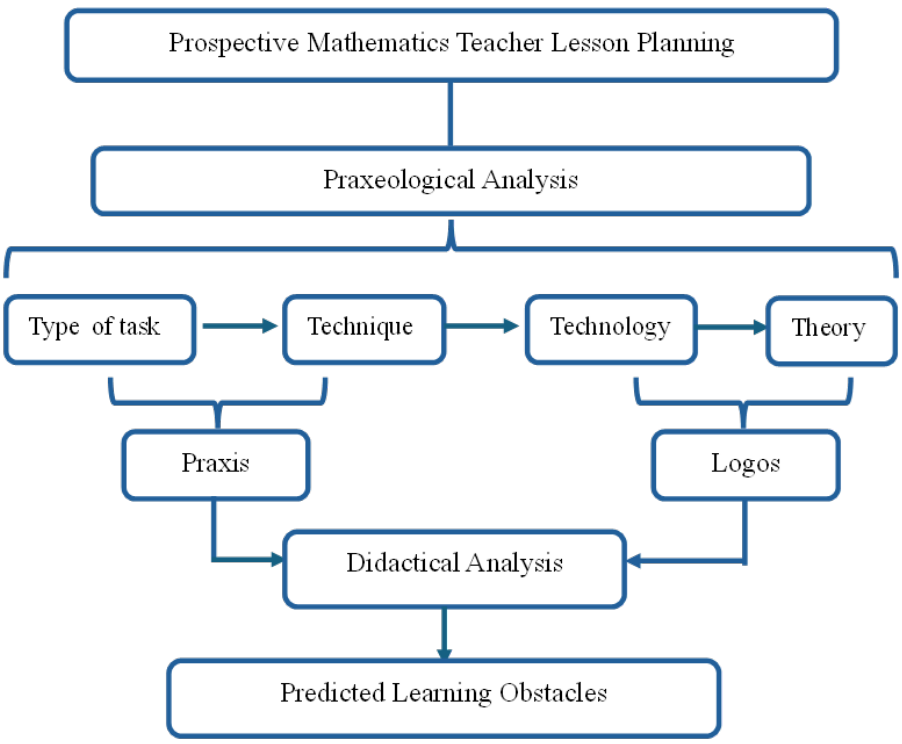


Figure 1. Praxeological Didactical Analysis Framework
(Adapted from (HastiYunianta et al., 2023))

Seven lesson plans were designed by prospective mathematics teachers during their participation in the school's introduction field course. They implemented those lesson plans in the classroom. The prospective mathematics teachers who take part in this program are those who have taken all courses related to teaching and mathematics content courses. To analyze in-depth, one of the 7 lesson plans was selected. The selection is based on the completeness of lesson plan components. Among them was implementing learning with a constructivist paradigm and the use of technology media in learning.

Results and Discussion

The results of this study include the mathematical praxiology of the lesson plans developed by prospective mathematics teachers as well as the prediction of learning obstacles that can be caused by the didactical design developed.

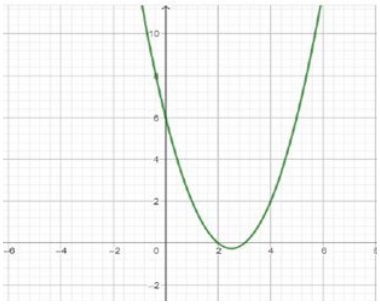
- *Praxeological Mathematics from Lesson Plans developed by Prospective Teachers*

To explain the results of the mathematical praxiology analysis of the lesson plans developed by the prospective teachers, we identified each task, justified the possible techniques students used to complete the task, the reasons students used the techniques, and the theory underlying the technology. The lesson plan developed by the prospective teacher who was the subject of this study contained several tasks, including: remembering the characteristics of a quadratic function graph, summarizing the steps of drawing a quadratic function graph, comparing the resulting graph image by Geogebra, and applying the steps of drawing a graph to a new problem. The mathematical praxiology in this lesson plan is written respectively in set notation $P_1(T_1, \tau_1, \theta_1, \Theta_1)$, $P_2(T_2, \tau_2, \theta_2, \Theta_2)$, $P_3(T_3, \tau_3, \theta_3, \Theta_3)$, and $P_4(T_4, \tau_4, \theta_4, \Theta_4)$. T is the mathematical task completed, τ do students use the predicted technique in completing the task, θ is an argument that supports the technique, and Θ is the theoretical basis that is considered in constructing mathematical concepts or procedures. Specifically, T_2 consists of sections including and $T_{21,} T_{22,}$ $T_{23,}$ dan T_{24}. Each contains a task with the same goal, namely constructing the steps for drawing the graph of a quadratic function. The mathematical praxiology is explained below.

The first mathematical praxiology, $P_1(T_1, \tau_1, \theta_1, \Theta_1)$, contains which facilitates students to remember the previous material learned, namely the characteristics of the function graph $f(x) = ax^2 + bx + c$. Students are also asked to explain the characteristics of a, b and c, if the graph of the function is known. According to the prospective teacher, the predicted technique on T_1 is that students observe the shape of the graph. If the curve opens upwards, then the value of $a > 0$, and if the curve opens downwards then the value of $a < 0$. Next, students observe the turning point on the curve. If the turning point is to the right of the y-axis then the value of $b > 0$, and if the turning point is to the left of the y-axis then the value of $b < 0$. Finally, determining the intersection point of the curve on the y-axis to determine the value of c. The mathematical praxiology P_1 is presented in the following table.

Table 2.

Praxeological Mathematics $P_1(T_1, \tau_1, \theta_1, \Theta_1)$

Mathematical Task (T_1)	Techniques (τ_1)	Technology (θ_1)	Theory (Θ_1)
Given the graph of a quadratic function. What are the characteristics of the following picture? a, b, and c? 	Predicted techniques: Looking at the shape of the graph, turning point, intersection of the curve with the y-axis and x-axis. Then determine the value of a, b, and c	The value of a, b, and c at $f(x) = ax^2 + bx + c$ can be determined by observing characteristics of the graph	• The upward open graph shows $a > 0$ • Downward open graph showing $a < 0$ • The position of the turning point determines the value of b • The point of intersection with the y-axis can be used to determine the value of c

Mathematical praxiology 2, $P_2(T_2, \tau_2, \theta_2, \Theta_2)$, contains tasks that guide students in answering the main problem, which is drawing the graph of a quadratic function. The tasks at include the steps to draw the graph of a quadratic function. The main problem to be answered in this task is as follows.

Perhatikan permasalahan di bawah ini!  Seorang pemain sepak bola terkenal Christiano Ronaldo menendang bola hingga membentuk sebuah lintasan : $-x^2 + 6x + 16$ Berdasarkan informasi yang telah diketahui, dapatkah kalian menggambar grafik fungsi kuadrat sesuai dengan lintasan tersebut	Translation according to the original: look at the problem below! A famous soccer player Christiano Ronaldo kicked the ball to form a trajectory: $-x^2 + 6x + 16$ Based on the known information, can you draw the graph of the quadratic function according to the trajectory.

Figure 2.

Problem Statement

To answer the above problem, 4 sub-praxeology are presented, namely $P_{21}(T_{21}, \tau_{21}, \theta_{21}, \Theta_{21})$, $P_{22}(T_{22}, \tau, \theta, \Theta)$, $P_{23}(T_{23}, \tau_{23}, \theta_{23}, \Theta_{23})$, and $P_{24}(T_{24}, \tau_{24}, \theta_{24}, \Theta_{24})$. So there are 4 sub-tasks on T_2, namely T_{21}, T_{22}, T_{23}, and T_{24}. T_{21} containing tasks to determine the intersection point of the graph with the x-axis. The predicted technique used by students is to determine the solution by factorizing the quadratic equation $-x^2 + 6x + 16 = 0$. The mathematical praxiology P_{21} is presented in the following table.

Table 2.

Praxeological Mathematics $P_{21}(T_{21}, \tau_{21}, \theta_{21}, \Theta_{21})$

Mathematical Task (T_{21})	Techniques (τ_{21})	Technology (θ_{21})	Theory (Θ_{21})
Determine the point of intersection of the graph with the x-axis. $y = -x^2 + 6x + 16$	Predicted techniques: Factoring and determining the solution of a quadratic function	The condition for determining the points of intersection with the x-axis is $y = 0$	The intersection point with the x-axis is obtained when y = 0.

The next praxeology, $P_{22}(T_{22}, \tau_{22}, \theta_{22}, \Theta_{22})$, contains the task of determining the point of intersection of the graph with the y-axis. The expected technique is that students substitute the value of x = 0 into the equation to obtain a point that intersects the x-axis. Mathematical praxeology P_{22} is presented in the following table.

Table 3.

Praxeological Mathematics $P_{22}(T_{22}, \tau_{22}, \theta_{22}, \Theta_{22})$

Mathematical Task (T_{22})	Techniques (τ_{22})	Technology (θ_{22})	Theory (Θ_{22})
Determine the point of intersection of the graph with the y-axis. Given: $y = -x^2 + 6x + 16$ $\Leftrightarrow y = -(0)^2 + 6x + 16$ $\Leftrightarrow y = \cdots$ the coordinates is …….	Predicted techniques: Verifying the given step.	The steps are given in detail.	The point of intersection with the y-axis is obtained when x = 0.

The next praxeology, $P_{23}(T_{23}, \tau_{23}, \theta_{23}, \Theta_{23})$, contains the task of determining the turning point or optimum point. The expected technique is to identify the values of and from the equation. Next, students substitute these values into the given formulas, namely the axis of symmetry and the optimum value formula, and then determine the coordinates. The mathematical praxeology P_{23} is presented in the following table:

Table 4.

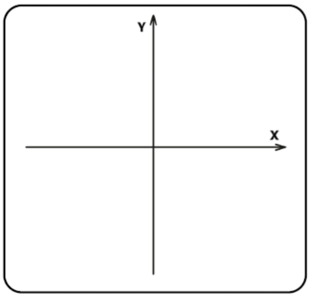
Mathematical Praxelogy $P_{23}(T_{23}, \tau_{23}, \theta_{23}, \Theta_{23})$

Mathematical Task (T_{23})	Techniques (τ_{23})	Technology (θ_{23})	Theory (Θ_{23})
Determine the turning point / optimum point $y = -x^2 + 6x + 16$. From the equation above: $a = \cdots, b = \cdots, c = \cdots$ Axis of symmetry: $X_p = -\frac{b}{2a} = \cdots$ Optimum value: $Y_p = -\frac{D}{4a} = \cdots$ The coordinates are…	Predicted techniques: Calculate X_p and Y_p	The steps are given in detail.	The optimum turning point is the pair of the axis of symmetry and the optimum value (X_p, Y_p)

$P_{24}(T_{24}, \tau_{24}, \theta_{24}, \Theta_{24})$ contains the task of plotting each point and then connecting the points with a line. This task presents a cartesian diagram, which displays an x-axis and a y-axis. The expected technique is to determine the position of the coordinates of the graph on the cartesian diagram provided and connect the points to form a curve. The mathematical praxiology P_{24} is presented in the following table.

Table 5.

Mathematical Praxeology $P_{24}(T_{24}, \tau_{24}, \theta_{24}, \Theta_{24})$

Mathematical Task (T_{23})	Techniques (τ_{23})	Technology (θ_{23})	Theory (Θ_{23})
Drawing a graph: Plot each point and then connect the points with a line 	Predicted techniques: Determine the coordinate position and connecting coordinate points to form a curve	Function graphs can be drawn by connecting the coordinate points that make up the graph, namely, intersection points to the x-axis, intersection points with respect to the y-axis, and turning points	The graph of a quadratic function is the set of points containing the intersection point on the x-axis, the intersection point on the y-axis, and the turning point.

The next praxiology, $P_3(T_3, \tau_3, \theta_3, \Theta_3)$, contains the task of checking the correctness of the graph drawn using the GeoGebra and provides a conclusion on the steps to draw a quadratic function graph. The mathematical praxiology P_3 is presented in the following table

Table 6.

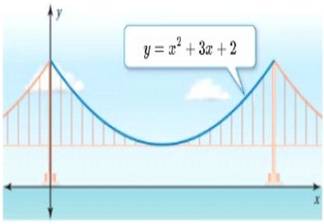
Mathematical Praxeology $P_3(T_3, \tau_3, \theta_3, \Theta_3)$

Mathematical Task (T_3)	Techniques (τ_3)	Technology (θ_3)	Theory (Θ_3)
Check the correctness : Is the graph you drew similar to GeoGebra? Can you summarize the steps to draw the graph of a quadratic function? Quadratic function?	Predicted techniques: Observe graphs presented by the GeoGebra application, compare it with the graph that students drew, and write down the steps of drawing a graph in the previous task	Function graphs can be drawn by connecting the coordinate points that make up the graph, namely, intersection points to the x-axis, intersection points to the y-axis, and turning points	The graph of a quadratic function is the set of points containing the intersection point on the x-axis, the intersection points on the y-axis, and the turning point.

The last praxiology, $P_4(T_4, \tau_4, \theta_4, \Theta_4)$, contains the task of determining the graph of a quadratic function on a different problem. The mathematical praxiology P_4 is presented in the following table

Table 7.

Mathematical Praxeology $P_4(T_4, \tau_4, \theta_4, \Theta_4)$

Mathematical Task (T_4)	Techniques (τ_4)	Technology (θ_4)	Theory (Θ_4)
Look at the picture below!  The rope bridge with the trajectory in the figure forms a curved curve, namely a parabolic curve. Parabolic curves are included in the graph of quadratic functions. Based on this information, can you determine: • Coordinates of the intersection point to the X-axis • Coordinates of the intersection point to the Y-axis • Turning point/optimum point • Draw the graph of a quadratic function	Predicted techniques: Follow the steps learned in the previous task	Follow the steps learned in the last task	The step of drawing a graph of a quadratic function is to determine the intersection point concerning the x-axis, the y-axis, and the turning point

Based on all the above mathematical praxeology, analyzed the thought process and its relationship, it can be concluded that its characteristics are as follows:

- The first praxeology describes a simple task that has the potential to facilitate students' use of previous learning experiences so that they can explain the characteristics of a quadratic function if the graph of the function is known. This situation provides students with a learning experience in converting representations from graphical to mathematical models.
- The second praxeology describes a math task opposite to the first task. That task asks students to draw a graph of a quadratic function. The second praxeology contains four steps. Students only draw the function's graph according to the steps given in the task.
- The third praxeology describes the use of the GeoGebra application to draw quadratic function graphs. Prospective teachers use it to verify whether the resulting graph is similar to the graph in the GeoGebra application.
- The fourth praxeology facilitates students' application of the procedure of drawing graphs in a new problem.

Students' experiences completing a series of interrelated tasks provide the characteristics of the math learning trajectory presented in the following figure.

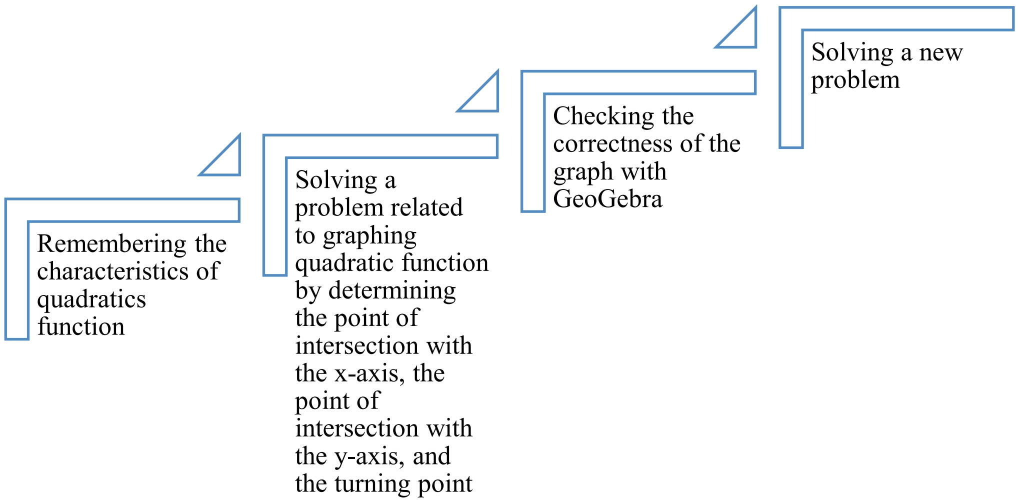


Figure 3. Learning Trajectory

- *Predicted Learning Obstacles from the Developed Didactical Design*

Learning is a process of producing knowledge. According to (de Grefte, 2023), knowledge is a true belief that includes three criteria: belief, truth, and justification. By the didactical situation theory put forward by Brousseau (2002), a didactic situation through action, formulation, validation, and institutionalization. The expected knowledge target in this learning design is to master the steps of drawing a quadratic function graph. That is, students build their knowledge about how to draw a graph according to their own learning experience.

The findings show that the praxeology $P_2(T_2, \tau, \theta, \Theta)$ contains tasks that guide students in concluding the steps of drawing graphs. Justification techniques (τ) done by students on T_{21}, T_{22}, T_{23}, and T_{24} are that students verify and answer according to the steps that have been given, namely determining the intersection point on the x-axis, determining the intersection point on the y-axis, determining the turning point, and plotting the point on cartesian coordinates then connecting the points to form a parabolic curve. So, in drawing graphs, students only follow the procedures determined.

Furthermore, $P_3(T_3, \tau, \theta, \Theta)$ contains T_3, which asks students to use the GeoGebra application to check the correctness of the graph. Using the GeoGebra application does not help students build their knowledge according to the expected learning objectives. Thus, overall, it can be concluded that the justification techniques in T_2 and T_3 are insufficient to help students construct the expected knowledge. This can lead to students not understanding why the steps are carried out. Students will rely on the mechanical procedures in each task without a deep conceptual understanding of the quadratic function, which can lead to errors in drawing graphs (López et al., 2016; Umah & Rahmawati, 2024).

Meanwhile, $P_4(T_4, \tau, \theta, \Theta)$ involves applying the knowledge already acquired to a problem similar to the previous problem. Presenting problems in similar and unvaried contexts will allow students to experience difficulties when there are transformations of the function graph, such as shifts or changes in scale, which require a more advanced understanding of how changes in the equation affect its graph (López et al., 2016).

Thus, the praxeology of P_2, P_3, and P_4 does not provide opportunities for students to construct knowledge and apply it to problems in higher contexts and difficulty levels. It allows students to struggle if they are given problems in a different context or difficulty level than those given in the examples (Reid O'Connor & Norton, 2024; Selowa & Dhlamini, 2023). According to (Brousseau, 2002; Hastiunianta et al., 2023; Suryadi, 2019), this can result in epistemological learning obstacles.

The diverse knowledge of students in a class is caused by differences in each student's previous learning experience. Based on Vygotsky's learning theory (L.S Vygotsky & Bakhtin, 1978; Prabowo & Juandi, 2020), students must start with something they already know to start learning. Thus, in designing learning designs, teachers must pay attention to students' skills and aptitudes in mathematics to facilitate students learning (Liiman et al., 2022). If the design does not facilitate the diversity of students' prior knowledge in the classroom, it can result in students experiencing learning barriers. For example, if the design is too difficult, students will experience obstacles in following the learning process. Conversely, if it is too easy, then student development will not be by the intellectual capacity that students should be able to achieve. One way to overcome the diversity of students' prior knowledge is to start learning by facilitating students to remember the prerequisite material that students must master.

In the material of drawing quadratic function graphs, students must have a prerequisite ability to draw simple algebraic function graphs and determine the position of points on cartesian coordinates (Siregar, 2017). It means that students first understand the meaning of function graphs before being developed to graph quadratic functions. This ability is important so that students do not experience misconceptions about the meaning of function graphs. When drawing graphs of simple algebraic functions, students may draw graphs by determining pairs of coordinate points according to the known domain first or by determining the important points that form the graph, such as the intersection point on the x-axis and the intersection point on the y-axis. Thus, the learning design facilitates students to use various understandings of function graphs before constructing how to draw a quadratic function graph. Meanwhile, the findings on the lesson plans designed by prospective teachers did not help students remember the prerequisite material. It indicates that the design can cause learning obstacles in students due to the diversity of students' prerequisite knowledge. These learning obstacles are called conceptual ontogenic obstacles (Brousseau, 2002; Suryadi, 2019)

One of the characteristics of mathematics is deductive reasoning, meaning that the truth of a concept or statement is obtained as a logical consequence of the previous truth so that the relationship between concepts or statements in mathematics is consistent (Hidayati, 2012). In presenting mathematical tasks, a task relates to the next task. Thus, the tasks presented must be in an orderly manner. The findings show that $P_1(T_1, \tau_1, \theta_1, \Theta_1)$ involves students' previous learning experience, namely about the characteristics of the quadratic function from the graph presented, but the next task at $P_2(T_2, \tau_2, \theta_2, \Theta_2)$ is for students to construct the steps of drawing a quadratic function graph. Thus, the characteristics of T_1 are not relevant to T_2. According to (Brousseau, 2002; Suryadi, 2019), learning design caused by the inaccuracy of the order of presentation of tasks in the design can cause didactic learning barriers.

Conclusion

The lesson plans developed by the prospective teachers under study facilitated students to be actively involved in representing from graphs to quadratic equations, inferring procedural steps in drawing graphs, and measuring understanding in performing these steps. The lesson plans indicated the existence of epistemology barriers, which are student barriers caused by certain context limitations. The presentation of tasks causes ontogenic obstacles due to inappropriate prerequisites. In addition, didactic obstacles are caused because there are tasks that are not relevant to the next task. The implication is that the results of the study can be used to improve a quality lesson plan, and the use of a PDA can be a consideration for prospective mathematics teachers when designing a quality lesson plan.

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