

Phenomenological Hermeneutic Study on the Epistemological Obstacles of High School Students in Solving Combinatorics Problems

¹Tri Nopriana, ²Sri Asnawati, & ³Nia Kania

^{1,2}*Universitas Swadaya Gunung Jati, Indonesia*

³*Universitas Majalengka, Indonesia*

¹trinopriana@ugj.ac.id

Abstract

This study employs a hermeneutic phenomenological approach to explore students' epistemological obstacles in solving permutation and combination problems, with the goal of supporting the development of more effective teaching materials. The research involved 24 12th-grade high school students (14 males and 10 females) and used qualitative methods. Data were collected through five diagnostic essay questions and semi-structured interviews to identify epistemological obstacles in combinatorics. Students' written responses and interview transcripts were analyzed and interpreted to uncover the underlying obstacles. The findings revealed several epistemological obstacles: (1) students were unable to identify all possible answers from a given problem; (2) students could not differentiate between problems requiring the concept of permutation and those requiring the concept of combination; (3) students struggled to solve problems that differed from the example problems provided; and (4) challenges in formulating a complete solution when faced with multiple conditions, despite being able to calculate partial results. These insights suggest that teachers and future researchers should consider students' epistemological obstacles when designing instructional materials, particularly for topics in combinatorics such as permutations and combinations. Developing learning resources based on these findings may enhance conceptual understanding and problem-solving skills among students.

Keywords: combination, epistemological obstacles, phenomenological hermeneutic, permutation, problem solving.

Introduction

Combinatorics is one of the essential mathematics topics for high school students. It is rich in problem-solving contexts, and solving problems related to counting principles requires deep mathematical reasoning, critical thinking, logical reasoning, and insightful understanding (Maher et al., 2011; Lockwood, 2013; Lockwood, 2015; Lockwood & Gibson, 2016). In addition, combinatorics is applied in various other fields such as computer science, communications, genetics, and statistics (Eizenberg & Zaslavsky, 2004; Lockwood, 2015).

The ability to solve combinatorial problems is not only essential for achieving mathematics learning objectives at the high school level, but it also serves as a stepping stone toward advanced mathematical learning, the development of logical thinking, and preparedness for real-world challenges. Several research findings indicate that a strong understanding of combinatorics can significantly enhance students' representational abilities and problem-solving strategies (Lockwood, 2013). English (2005) emphasizes that combinatorial skills play a critical role in fostering systematic thinking and strategic flexibility in solving complex

problems. Studying combinatorics helps students develop skills in enumeration, prediction, generalization, and systematic thinking (Kapur, 1970).

However, research reveals that students face difficulties with permutation and combination problems, with permutations often considered more challenging (McGalliard, 2012). Wasserman (2013) found that students can better grasp the structural characteristics of permutation problems (ordered choices) than combination problems (unordered choices). Frequent mistakes in applying the formulas suggest students struggle to understand when and why these formulas are used (Batanero et al., 1997; Lockwood, 2011; Nusantara & Chandra, 2016).

Two steps to teaching permutation and combination material easily are to understand students' difficulties in solving combinatorics problems and identify variables that may be the cause of these difficulties (Batanero et.al, 1997; Lockwood, 2013). Students' mathematical ability is influenced by internal factors and their surrounding environment (Marticion, 2021). Understanding and identifying students' difficulties in solving enumeration rule problems is done by analyzing learning obstacles. Learning obstacles are pieces of knowledge, not difficulties or lack of knowledge (Brousseau, 2002). These pieces of knowledge can be used in familiar contexts, but in different contexts, this knowledge cannot be used, thus creating obstacles. One type of learning obstacle according to Brousseau (2002), is epistemological obstacles. Epistemological obstacles are learning obstacles that are formed because some of the knowledge that occupies the conceptual structure is correct in a certain context, but cannot be applied in a new context (Suryadi, 2019). So to achieve success in learning permutations and combinations, it is necessary to analyze the learning obstacles experienced by students in solving permutation and combination problems.

Some findings related to students' learning obstacles in solving permutation and combination problems based on relevant research results include errors in understanding the concept of permutation and combination (students are confused about which cases use the concept of permutation, which are combinations), errors in compiling mathematical models, errors in using concepts in solving problems, and procedural errors, resulting in calculations that are larger than the desired results, difficulties in choosing the right operation and writing factorial signs in permutations and combinations (Lockwood, 2011; Lockwood & Gibson, 2016; Rahayuningsih, 2016; Astuti, 2017; Dwinata & Ramadhona, 2018; Meika & Suryadi, 2018; Rizqika et.al, 2019). Furthermore, students are still confused about solving permutation problems in simple forms and are unable to solve complex problems, are unable to solve combination problems with various variations, are unable to distinguish between the use of permutations and combinations in problems, students do not make a complete solution plan (Mahyudi, 2016; Wahyuniar & Widyawati, 2017).

The obstacles encountered by students in solving permutation and combination problems in previous studies were identified among high school students and were mostly explored using basic qualitative approaches. In contrast, this study examined epistemological obstacles through a phenomenological approach known as hermeneutic phenomenology. A phenomenological study describes the general meaning for several individuals from their life experiences of a concept or phenomenon (Cresswell, 2013: 76). Meanwhile, hermeneutics is a theory and methodology of interpretation, especially from texts. Hermeneutics is concerned with the process of interpretation and how meaning can be expressed and understood. So to

conduct hermeneutic phenomenological research requires the ability to examine the text, and reflect on its contents to find something meaningful (Van Manen, 1990). Based on the description of the importance of analyzing learning obstacles for high school students and the importance of using the hermeneutic phenomenology approach in interpreting the results of the analysis of student learning obstacles. Therefore, this study aims to reveal the epistemological learning obstacle of high school students in solving permutation and combination problems, using the hermeneutic phenomenology approach. Therefore, the research questions to be answered in this study is: What are the epistemological learning obstacles experienced by high school students in solving permutation and combination problems, as revealed through a hermeneutic phenomenology approach?

Methods

This study employed a Didactical Design Research (DDR) design (Suryadi, 2019a). DDR was a research approach aimed at identifying learning obstacles, including epistemological obstacles, in the learning process and was intended to anticipate and eliminate these obstacles (Suryadi, 2019b). To uncover epistemological obstacles, the researcher used a hermeneutic phenomenology approach. Hermeneutic phenomenology was a type of phenomenological research that emphasized both describing and interpreting the lived experiences of individuals (Creswell, 2013:76). The main goal of phenomenological research was to reduce individuals' experiences of a phenomenon into a description that captured the true essence of the subject (Van Manen, 1990:177). Therefore, in this study, the hermeneutic phenomenology approach was used to explore the experiences of high school students in solving combinatorics problems. The researcher then interpreted these experiences based on students' written responses, documentation, and interview results. This study involved 24 twelfth-grade students, consisting of 14 male and 10 female students, from a public high school in Cirebon City. Analyzing students' epistemological obstacles was part of the prospective analysis stage in Didactical Design Research (DDR) (Suryadi, 2013). The instruments used included five essay questions on permutation and combination topics, as well as interview guidelines to confirm the epistemological obstacles experienced by the students. The data analysis technique followed the phenomenological method described by Creswell (2013:193), which consisted of the following steps: (1) students, as research participants, described their personal experiences related to learning permutations and combinations; (2) the researcher identified statements (from interviews or other data sources) about the learning obstacles students encountered; (3) significant statements were extracted and grouped into broader units of information, known as "meaning units" or themes; (4) the researcher wrote a description of "what" the students and teachers experienced; (5) the researcher then wrote a description of "how" the experience occurred—this is referred to as the "structural description", where the researcher reflects on the setting and context in which the phenomenon was experienced; (6) finally, a composite description of the phenomenon was developed, combining both the textural and structural descriptions to capture the essence of the experience.

Results and Discussion

Several types of epistemological learning obstacles that were found were grouped based on the problem being presented as follows.

Epistemological learning obstacles in solving problems 1

Problem 1.

It is known that there are 5 routes from Cirebon to Bandung, 6 routes from Bandung to Jakarta, and 2 routes from Cirebon to Jakarta. The number of outbound and return routes between cities is the same. Andi and Rizki were asked to count the number of different ways to go from Cirebon to Jakarta and return to Cirebon without going through Bandung. Andi answered 60 ways, while Rizki answered 64 ways. Whose answer is correct? Explain the reason!

60 cara
mengapa?
dik: Cirebon - Bandung = 5 rute
Bandung - Jakarta = 6 rute
Jakarta - Cirebon = 2 rute
Jawab: $5 \times 6 \times 2 =$
 $30 \times 2 = 60 \text{ rute}$

Figure 1. Epistemological Obstacle of Student 1 in Solving Problem 1.

In this problem, students are asked to be able to re-check the results of the calculations that have been known regarding the number of travel routes from Cirebon to Jakarta and vice versa. Student responses in solving this problem are grouped into 2 types. The first type is the response from student 1 (Figure 1.) and student 2 (Figure 2.). The second type is the response from Student 3 (Figure 3.) and Student 4 (Figure 4.).

In solving problem 1, these two high school students solved the route problem using the multiplication rule. As seen in Figure 1, student 1 first wrote down the information contained in the problem. At this stage, students are considered to have been able to understand the problem by identifying the information contained in the problem. Students try to analyze the number of intercity routes and multiply them. To find out more about the obstacles that students experience in solving this problem, the researcher interviewed the students. Initially, the researcher asked how the students solved problem 1. The students gave responses similar to those written on the answer sheet. This shows that the students have correctly understood the answers written on the student worksheet. Next, the researcher asked the students to check if there were other ways to get to Jakarta. After being asked this question, the students just thought that there was another way to get to Jakarta, namely directly from Cirebon to Jakarta without going through Bandung. Student 1 explained that there were 2 ways, the first as student 1 had written on the answer sheet, and the second was a direct route from Cirebon to Jakarta. The following is an excerpt from the interview conducted with student 1.

R: Try calculating, from Cirebon directly to Jakarta there are 2 routes, right? There are 2 routes for departure and return. So how is it?

S1: Oh yeah, like before, multiply it, ma'am. So $2 \times 2 = 4$, ma'am. So the answer is 64, right, ma'am, Rizki?

R: Why is it 64?

S1: Hehe, if Andi is wrong, Rizki is right, plus that, ma'am, the route 60 plus the 4.

The summary of the interview conducted by the researcher with student 1 shows that the student missed or did not think that there was another way to get from Cirebon to Jakarta. The student did not calculate the other way, but with a little help, student 1 was able to calculate the possibility of another route. However, the next problem was that when he had calculated another way to Jakarta, student 1 did not know why the two possibilities had to be added together. The student only guessed, if Andi's calculation was wrong, then the correct one was Rizki's calculation.

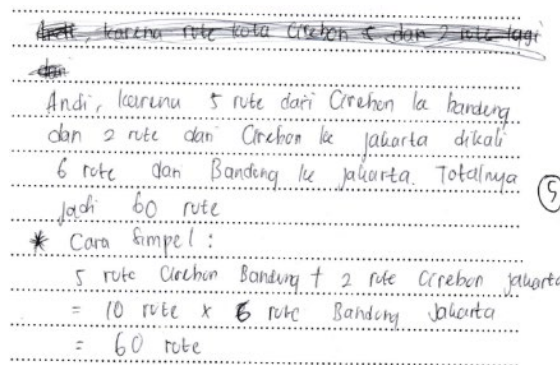


Figure 2. Epistemological Obstacle of Student 2 in Solving Problem 1

Next, the answer of student 2 was different from student 1. Student 2 explained that in solving problem 1, he multiplied the route from Cirebon to Bandung, Bandung to Jakarta, and the return from Jakarta to Cirebon. The student's explanation did not match the answer given on the answer sheet. Therefore, the researcher reconfirmed with the student as documented in the following interview excerpt.

S2: Oh yes ma'am, that was wrong, I should have multiplied 5 by 6 then multiplied by 2 ma'am. I just wrote 5 by 2. Then I forgot the 6. So multiplied by 6.

Next, the researcher asked whether there was another way to get to Jakarta. Student 2 felt that there was no other way and his answer was correct. However, after the researcher asked the student to read it again, the student found that to go to Jakarta, it could be taken directly from Cirebon. However, in determining the final number of all routes, the student did not know why it was necessary to add up the two possible routes. Student 2 only mentioned that if multiplied again, the number would be too large. In this case, there is an epistemological obstacle for students, when they have been able to calculate all possible routes that can be taken, but students have difficulty in determining the next process in solving the problem. This is related to students' understanding of the rules of multiplication and addition. Students do not yet know when the concept of the rules of multiplication and addition is used. Other responses given by students in solving Problem 1 are explained in Figure 3 and Figure 4 below.

$$\begin{aligned}
 {}^nC_r &= \frac{n!}{(n-r)!r!} \\
 {}^{13}C_2 &= \frac{13!}{(13-2)!2!} = \frac{13!}{11!2!} = \frac{10 \times 11!}{11!2!} \\
 &= \frac{1 \times 2 \times 3}{1!1 \times 2} \\
 {}^{13}C_2 &= \frac{3}{1}
 \end{aligned}$$

Figure 3. Epistemological Obstacle of Student 3 in Solving Problem 1.

Students 3 and 4 gave similar responses, namely solving the problem directly using the permutation or combination formula. The researcher first asked the students to explain again what they wrote on the answer sheet. Student 3 stated that n indicates the total routes that are equal to 13 ($5 + 6 + 2$) and r which states the number of cities is equal to 3 (Cirebon, Bandung and Jakarta). The reason why students immediately used the combination formula is explained in the following interview conversation excerpt.

S3: If I'm not mistaken, the combination does not pay attention to the order, ma'am, so the route problem can be various, ma'am, so I use a combination.

P: Okay, so why did you enter the formula directly?

S3: Well, if the problem is certain, if it's not a permutation, it must be a combination, ma'am.

Student 3 (Figure 3.) uses the combination rule, while Student 4 uses the permutation rule as presented in Figure 4. below.

$$\begin{aligned}
 n &= 5 & P_k^n &= \frac{n!}{(n-k)!} \\
 k &= 2 \\
 P_{2,5} &= \frac{5!}{(5-2)!} \\
 &= \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} \\
 &= 60 \text{ cara}
 \end{aligned}$$

Jawaban Andi karena semakin sedikit cara yang
tapi jawaban paling banyak benar.

Figure 4. Epistemological Obstacle of Student 4 in Solving Problem 1

Similar to student 3, student 4 also immediately applied the formula in solving problem 1. The researcher dug up information about the reasons why students used the permutation formula and an explanation of the values of n and k that student 4 wrote on the answer sheet. The following is an excerpt from the researcher's interview with student 4 in solving problem 1

S4: This is a route, ma'am, so it's a permutation, ma'am. If it's a combination, usually it doesn't pay attention to the order. If this is the route, it has to be Cirebon to Bandung and then to Jakarta. So we use the concept of permutation, ma'am.

P: So why is n equal to 5 and k equal to 2?

S4: Well, I'm guessing, ma'am. I calculated the result that was around 60. If k, the route is 2, ma'am when going and coming back.

The results of the interview excerpts with Students 3 and 4 showed that when faced with problems related to enumeration rules, students tended to immediately use formulas. Although students were able to provide arguments as to why they used the permutation or combination formula, after continuing with the next question, students experienced obstacles in understanding the problem and using the concept of permutation or combination. Students felt that if given a math problem, it was always related to a math formula. Students often tend to use formulas or procedures that have been learned without considering the context of the problem as a whole and understanding the underlying mathematical context (Verschaffel, et.al., 1999; Lesh & Harel, 2003). Focusing too much on the use of formulas or algorithms in mathematics teaching can reduce students' ability to solve problems creatively or to develop a deep understanding of fundamental mathematical concepts (Schoenfeld, 1985).

In discussion, it is the most important section of your article. Here you get the chance to sell your data. Make the discussion corresponding to the results, but do not reiterate the results. Often should begin with a summary of the main scientific findings (not experimental results). The following components should be covered in discussion: How do your results relate to the original question or objectives outlined in the Introduction section (what)? Do you provide interpretation scientifically for each of your results or findings presented (why)? Are your results consistent with what other investigators have reported (what else)? Or are there any differences?

Epistemological learning obstacles in solving problems 2

Problem 2.

Three students from grades I, II, and III will be elected as three members of the OSIS board. The OSIS board consisting of three people must be formed and each class must be represented by a student. The following are the names of the candidates for the board from each class.

<i>Class Administrator</i>		
<i>Grade I</i>	<i>Grade II</i>	<i>Grade III</i>
<i>(D)</i>	<i>(T)</i>	<i>(A)</i>
<i>(N)</i>	<i>(I)</i>	<i>(R)</i>
<i>(G)</i>	<i>(S)</i>	<i>(B)</i>

- Based on the given conditions, can the composition of members consisting of (D), (R) and (A) become the administrators of the Student Organization? Provide an explanation!*
- Create a mathematical model to calculate the number of ways to determine the composition of Student Organization management that may be formed!*

Overall, students can solve problem 2 well. In answering part a of question 2, all students can answer correctly. This shows that students can understand the problem well. Students understand that there is a rule that each class must delegate one student. This means that students understand that no class can delegate 2 people. R and A are from class III. So the composition of the management cannot be the management of the Student Organization. Furthermore, when answering part b, most students have been able to write a mathematical

model well as seen in the following picture. However, there is a finding obtained from the response of one of the students. The finding can be seen in the following explanation.

- b. Buatlah model matematika untuk menghitung banyaknya cara menentukan susunan kepengurusan OSIS yang mungkin dibentuk!

The image shows a student's handwritten work on a lined paper. The top line shows a calculation that has been crossed out with a single horizontal line: $P(9,3) = 9! (9-3) = (9 \cdot 8 \cdot 7 \cdot 6) 6 = 9 \cdot 8 \cdot 7$. Below this, the student has written the permutation formula: $P_r^n = \frac{n!}{(n-r)!}$ or $n! (n-r)!$. To the right of the formula, there is a circled number '1'.

Figure 5. Epistemological Obstacle of Students in Solving Problem 2

The next type of epistemological obstacle experienced by high school students in solving enumeration rule problems can be seen in the answer of one of the students in part b, when the student was asked to write a mathematical model to solve the problem, the student answered that the mathematical model was by using the concept of permutation. Initially, the student had correctly described the permutation formula by writing $P_r^n = \frac{n!}{(n-r)!}$. However, the student wrote again that the permutation formula can also be written as $n! (n-r)!$. The student assumes that the mathematical sentence $\frac{n!}{(n-r)!}$ can also be written as $n! (n-r)!$. This is confirmed by the following interview transcript.

R: To answer part b, why did you write two permutation formulas? Can you explain?

S: Yes ma'am, the permutation formula is like that ma'am, $P_r^n = \frac{n!}{(n-r)!}$. This formula can also be written ma'am $P_r^n = n! (n-r)!$.

R: Why can it be written like that?

S: Yes ma'am, if I remember correctly ma'am, because it's difficult to divide ma'am, so it can be written with multiplication like that.

This shows that high school students have difficulty understanding mathematical notation. Permutations and combinations often use complex mathematical notation such as nCr and nPr . Students who are not familiar with this notation may have difficulty understanding and using formulas related to permutations and combinations. Based on the findings of the research results, it was found that students assume that the mathematical model $\frac{n!}{(n-r)!}$ can also be written as $n! (n-r)!$.

Epistemological learning obstacles in Solving problems 3

Problem 3.

In the midterm exam for mathematics, there are 15 questions consisting of 10 multiple-choice questions and 5 essay questions. Each student is only required to work on 8 multiple-choice questions and 2 essay questions. Count the number of ways to choose the questions!

Diket: n soal pilihan ganda = 10
 r soal pilihan ganda = 8
 n soal essay = 5
 r soal essay = 2

Kombinasi soal PG = $\frac{n!}{(n-r)!r!} = \frac{10!}{(10-8)!8!} = \frac{10 \times 9}{2!}$
 $= \frac{90}{2} = 45 \text{ cara}$

Kombinasi soal essay = $\frac{n!}{(n-r)!r!} = \frac{5!}{(5-2)!2!} = \frac{5 \times 4 \times 3}{3!}$
 $= \frac{5 \times 4 \times 3}{3 \times 2 \times 1}$
 $= \frac{60}{6} = 10 \text{ cara}$

Jadi cara memilih soal pilihan ganda adalah = 45
cara, sedangkan memilih soal essay adalah
10 cara

Figure 6. Epistemological Obstacle of Students in Solving Problem 3

The epistemological obstacle encountered by students in solving problem 3 is that students do not know how to solve the given problem properly. It can be seen from Figure 4.17 that students have been able to calculate the combination of multiple-choice and essay questions. However, when writing the conclusion, students write down each way of choosing questions. Students do not use the concept of multiplication rules. Students still write them separately. Some information about the obstacles experienced by students is explained in the following excerpt from the student's interview with the researcher.

S: Yes ma'am, I was confused about what to do after that, I wanted to multiply it but I was afraid of making a mistake and I wanted to add it up but I was also afraid of making a mistake.

P: So should each result be multiplied or added up?

S: It seems like it should be added up, ma'am. They did multiple-choice and essay questions. So both questions were answered

Based on the interview results, it was obtained that the student knew that multiple choice and essay questions needed to be done together. However, the student stated that the number of possibilities for working on multiple-choice and essay questions should be added up. This shows that students experience epistemological obstacles in understanding the addition and multiplication rules in the enumeration rules material.

The results of Nopriana et.al's (2023) research show that the epistemological obstacles that occur in students in solving permutation and combination problems include: (1) difficulty in constructing sentences/mathematical models; (2) difficulty in determining the concept of permutation and combination used; (3) difficulty in finding solutions to problems that have never been encountered before; (4) errors in understanding previous concepts (namely the rules of addition and multiplication). Students still find it difficult to solve problems when it comes to the rules of multiplication and addition. This may happen because students are too focused on memorizing permutation and combination formulas. If they only memorize formulas or procedures without understanding the basics, they may have difficulty when faced with more complex or unfamiliar problems. A strong understanding of the rules of multiplication and

addition in the counting rules requires continuous practice. Students need to have a deep understanding of these concepts and gain skills in applying them. Some students may have difficulty understanding the logical steps involved in solving counting rule problems. Solving mathematical problems often involves solving the correct sequence and understanding the relevant steps. If students cannot follow the logical sequence or understand the relationship between the different steps, they may feel stuck or confused in solving the problem.

Students may have difficulty seeing the relevance of the rules of multiplication and addition in everyday life. When students cannot make connections between mathematical concepts and real-world situations, they tend to lose interest and motivation to understand them well. Therefore, it is important to provide concrete and relevant examples that help students see how the rules of multiplication and addition can be applied in everyday life.

Several epistemological obstacles encountered by students were found when the researcher conducted further interviews after the students had solved the given permutation and combination problems. In a phenomenological hermeneutic study, students can freely speak and expand the discussion without interrupting. The researcher in this case would summarize, rephrase, probe, ask follow-up questions and whether there was anything further (Dahlberg & Dahlberg, 2020). Some of the students' responses related to the questions given are explained as follows:

Question 1: What difficulties did you experience while working on the given problem?

Student 1: If the problem is different from the example, I am confused and cannot do it.

Student 2: Confused, ma'am, when to use the combination formula and when to use the permutation formula.

Student 3: If there is a question like before, ma'am, who chooses multiple choice and essay questions, then after that, ma'am, I'm confused about what to do. And one more thing, ma'am, when calculating the route, there was something that was missed, ma'am.

Interview excerpts conducted by researchers with student 1 show information that obstacles are experienced in solving problems that are different from examples that have been studied before. Students provide statements that the problem of routes and how to choose questions has never been discussed before. Students often face difficulties when the questions given are different from examples that have been studied because they tend to rely on patterns and examples that they have seen before. When new problems do not fit the pattern, they may not know how to start or apply the concepts they have learned. This is related to the learning transfer process that has not been perfectly carried out in learning. Learning transfer or the ability to apply knowledge and skills learned in one context to another context is often an obstacle to learning (Bransford, Brown & Cocking, 2000). In general, according to cognitive load theory, students tend to build specific knowledge schemes based on examples studied (Sweller, 1988). When they are faced with problems that do not fit this scheme, they experience a high cognitive load that inhibits problem-solving. Based on the theory of didactic situations (Brousseau, 2002), Student 1 has not mastered the institutionalization situation where students use the knowledge of learning outcomes and apply it to different contexts (different other problems).

Furthermore, Student 2 experienced confusion in distinguishing problems between situations that require the use of permutation and combination formulas. This learning obstacle can be caused by a conceptual understanding that is not deep enough about when and how to

use each formula. This is following Lockwood's research (2013) which states that students often experience confusion in choosing and using the right formula due to a lack of conceptual understanding of permutations and combinations. They tend to memorize formulas without understanding the context of their application. In addition, this may also be caused by instructions in the problem that are inadequate or difficult for students to understand. Instructions that focus too much on procedures rather than conceptual understanding can cause difficulties in distinguishing between permutation and combination situations (English & Warren, 1995).

Student 3 stated that obstacles in solving the problems given include determining the conclusion of the solution to the problem given. This relates to students' understanding of the rules of multiplication and addition. In the discussion of didactic obstacles, it has been conveyed that teaching materials have not been completely prepared, especially in conveying basic calculation rules including multiplication and addition rules. According to Rittle-Johnson, Siegler, & Alibali (2001), students who only have procedural understanding tend to have difficulty applying mathematical rules in new contexts. Research by Jonassen (2000) shows that problems that require the application of multiplication and addition rules in more complex contexts often confuse students. They may be able to perform calculations individually but have difficulty integrating the results of the calculations to draw the right conclusions. The results of the study indicate that deep conceptual understanding and appropriate instruction are essential in overcoming this learning barrier. Teaching that focuses on understanding context and applying concepts can help students overcome these obstacles and improve their ability to solve complex mathematical problems.

Hermeneutic phenomenological studies have also been employed by several researchers to interpret and explore students' experiences, particularly in understanding the learning difficulties they face, including challenges with non-routine number patterns (Aiyub, et.al, 2024), develop meaning fraction and fractional multiplication (Isnawan, et.al, 2022; Isnawan, et.al, 2023). Epistemological obstacles refer to the difficulties or obstacles faced by students in acquiring, understanding, and applying certain knowledge or concepts. Epistemological obstacle caused by the limitations of certain context (Musyrifah, et.al, 2024). In the context of High School students who solve enumeration rule problems, epistemological obstacles can include various factors that hinder the learning process and understanding of related mathematical concepts.

Based on the research findings presented, this study addresses the needs of mathematics teachers, particularly in identifying the epistemological obstacles experienced by high school students through a hermeneutic phenomenological approach. This approach emphasizes the interpretation of students' learning experiences in studying permutations and combinations, specifically through their written responses and interview data. It is hoped that by identifying the epistemological obstacles that arise, the design of instructional materials can be developed to help overcome these obstacles effectively.

This study has several limitations. First, the research was conducted with a relatively small number of participants from selected high schools, which may limit the generalizability of the findings to broader student populations. Second, as a qualitative study using a phenomenological hermeneutic approach, the results are interpretive in nature and rely heavily on the depth and clarity of student responses during interviews and observations. This means

that certain cognitive processes or obstacles may remain unexplored if they were not explicitly articulated by the participants. Third, the focus was limited to epistemological obstacles within combinatorics problems, so other forms of learning obstacles—such as didactical or ontogenic—were not examined in depth.

Conclusion

The results of the study indicate that the study of hermeneutics phenomenology is used to interpret students' experiences in solving permutation and combination problems. Students experience epistemological obstacles in solving permutation and combination problems that are seen based on the results of document studies of student work in the form of text and interview results. Several findings related to epistemological obstacles that emerged include (1) Students have not been able to determine all the possibilities that occur, some possibilities are not calculated; (2) Students cannot distinguish between permutation or combination conditions given in the problem; (3) Students have difficulty in solving problems that are different from the examples given by the teacher; (4) Students still find it difficult to solve problems when they are related to multiplication and addition rules; (5) When there are 2 conditions, students can calculate each possibility, but have difficulty in completing the final part of the problem related to multiplication and addition rules. To help students overcome epistemological obstacles, teachers need to design a permutation and combination teaching material design that can accommodate the learning obstacles that arise. The teaching materials need to contain didactic situations that allow students to re-check the final answers, and achieve institutionalization situations so that students can differentiate between permutation and combination problems, and students can solve problems that differ from the examples given

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