Examples of the Use of the Scientific Approach in Mathematics Teaching and Learning to Help Indonesian Students to be Independent Learners

Fadjar Shadiq
Yogyakarta, Indonesia
fadjar_p3g@yahoo.com

Abstract

This is a theoretical paper focusses on Indonesian school system. The challenge for education in Indonesia according to the former Minister of Education and Culture of Indonesia, Anies Baswedan, was how to help Indonesian students to be independent learners and to have good characters (*Kemdikbud*, 2014). The 2013 Curriculum proposed Scientific Approach to be implemented in Indonesian mathematics classes. Scientific Approach consists of five steps: (1) observing, (2) questioning, (3) collecting data, (4) reasoning, and (5) communicating. This paper discusses how two approaches, namely Scientific Approach and the Japanese Problem-solving Approach (PSA), can help Indonesian students to improve their thinking, creativity, and innovation during mathematics teaching and learning in classroom. The paper will provide some practical examples of problem-solving using these two approaches.

Keywords: Japanese Problem-solving Approach, Scientific Approach, problem posing, independent solving, observing, questioning, reasoning.

Introduction

The University of Tsukuba in Japan has successfully produced three Nobel laureates. They are Dr. Tomonaga Sin-Itiro in Physics (1965), Dr. Esaki Leo in Physics (1973), and Dr. Shirakawa Hideki in chemistry (2000) for his invention and development of conductive polymers (University of Tsukuba, 2012). This could also happen in Australia, USA, UK, and China. Those Nobel laureates have made an impact on the development of industry, economy, and technology in the country. Thus, some people are the leaders in our world and not just the followers. Now, can you imagine if that university with three Nobel laureates is from Indonesia or from other SEAMEO Member Countries? This should be a goal not only for mathematics teachers and educators but also for all Indonesian. Therefore, the following questions might arise: How can they achieve such a high level? Can we do it? What do we need to change? What should we learn? Can mathematics education support our dream?

Discussing these questions with mathematics teachers and educators from SEAMEO Member Countries who attended the courses in SEAMEO Regional Centre for QITEP in Mathematics (SEAQiM) is always challenging. From the discussion, at least three things were identified. First of all, success was related to the characters, attitudes, and spirit of life of the Japanese. In general, the participants of the courses concluded that the Japanese want to be the best and tough. Japanese culture includes traditions such as *bushido* and *samurai* that contribute to their success. Second, the Japanese have the capabilities to explore, investigate, experiment, discover, and inquire. These capabilities are related to thinking, reasoning, and process skills. Lastly, the Japanese exhibit competence in content knowledge of each school subject.

The challenge for education in Indonesia is how to help Indonesian students to be independent learners and to have good characters (Kemdikbud, 2014). Thus, there are two problems needed to address, namely (1) students' thinking, creativity and innovation and (2) students' character (Shadiq, 2014).

Shadiq (2016) stated that Indonesian students should learn mathematics meaningfully and joyfully by emphasising thinking and reasoning. This requires teachers to change and improve the quality of the teaching and learning process from a "typical" or "traditional" mathematics classroom to a more innovative one (Shadiq, 2010). Activities in classroom mostly stress memorizing and do not encourage students to think, to reason, and to innovate. Teachers still use the paradigm of transferring knowledge from their brain to students' brain. Another alternative of mathematics activities leans more toward exploration, inventing, discovering or experimenting of mathematical concepts through conceptual investigation or exploration. Students might use concrete materials, such as manipulative both concrete and virtual, and participate in experiments and kinaesthetic demonstrations that exhibit mathematical concepts. This type of mathematics activities matches constructivism, one of the current trends in education. Haylock and Thangata (2007) stated that constructivism focuses attention on the students' learning rather than the teacher's teaching.

Another focus in mathematics education is the use of discovery, Bruner (Cooney, Davis, & Henderson, 1975) stated that learning by discovery is learning to discover. Problem-solving is learning to solve problem, similarly exploration, investigation, and experimentation is learning to explore, to investigate, and to experiment. Thus, it is clear that during mathematics teaching and learning process, students should be facilitated to develop their thinking ability, creativity, and innovation so that they can apply their knowledge, skills, and attitudes in real life situations beyond school. Isoda (2015a, 2015b) summarised this by asserting that teachers should develop children to make use what they have learned before without teachers support. If they have developed then they can reply to the question: What do you want to do next?

Why it is so hard to change the way teachers teach, Goos, Stillman, and Vale (2007, p. 4) stated "Whether we are aware of it or not, all of us have our own beliefs about what mathematics is and why it is important." Furthermore, they cited Barkatsas and Malone (2005) who stated:

> Mathematics teachers' beliefs have an impact on their classroom practice, on the ways they perceive teaching, learning, and assessment, and on the ways they perceive students' potential, abilities, dispositions, and capabilities (Goos, Stillman & Vale, 2007, p. 71).

There is a need to address not only the content knowledge and pedagogical practices of teachers but also their beliefs and attitudes to change real teaching practice. Does the SA contribute to the changing process (Shadiq, 2016a, 2016b & 2018)? Can the SA be applied to help and facilitate Indonesian students to improve their thinking ability, creativity, and, innovation during mathematics teaching and learning?

In Japan, Isoda and Katagiri (2012) proposed a different model called Problem-solving Approach (PSA) which consists of four steps: (1) problem posing, (2) independent solving, (3) comparison and discussion, and (4) summary and integration. These two will be investigated further in the paper.

The process of learning mathematics in the classroom will be largely influenced by the beliefs of the mathematics teacher towards mathematics itself. Therefore, imperfections in understanding of mathematics will in some ways lead to imperfections in teaching and learning process. In other words, the correct beliefs and understandings of mathematics are expected to help the process of mathematics teaching and learning be more effective and efficient to meet students' needs.

What is Mathematics?

Formulating the definition of mathematics is not as easy as imagined. This is because the definition is influenced by the purpose of mathematics teaching and learning in the classroom and the adjustments made for the changing of students' needs. Mathematics should be used to help and facilitate students so that they can compete with other citizens. In the past, for example, mathematics was defined as a study of numbers or shapes. Therefore, many mathematics teachers and educators focused on skills and mostly used procedural practice. This instruction focused on memorization and skill-and-drill practice where teachers offered lecture type instruction and students completed the pages in the texts during class time.

Now, there is a growing demand where mathematics learning should be more inductive and not always deductive. Therefore, students will learn to digest new ideas, adjust to change, handle uncertainty, find regularity, and solve unusual problems. The definition that fits is mathematics is science that discusses patterns or regularities. Mathematics learning goals that have been established by the MoEC and are in accordance with the latest trends need to get supported by all parties. Thus, mathematics examination should support the achievement of learning objectives in the classroom.

To answer the question 'What is Mathematics?' it is important to learn from a young Gauss (1777-1855) in solving the problem of: $1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100$. Gauss was known widely as one of the five best mathematicians around the world. When he was 10 years old, his teacher asked him and his friends to find the result of: $1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100$. His teacher intended that the class used a "typical" or "traditional" method. In this case, students should find the sum of 1 and 2, that is, $1 + 2 = 3$, then calculate $3 + 3 = 6$, continue with $6 + 4 = 10$, and so on until finally find the sum of the last sum with 100. How long does it take to find $1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100$ by using this method?

A young Gauss applied different method. He observed patterns as shown in Figure 1. Gauss found that by pairing numbers from each end of the sequence gave the same sum: $1 + 100 = 101$, $2 + 99 = 101$, $3 + 98 = 101$, and so on. The young Gauss concluded, as part of his reasoning activity, that: (1) every number has a pair such that the sum of each pair was 101; (2) there were 100 numbers to be added, implying that there were 50 pairs altogether of two numbers that each sum was 101; and (3) the result of addition $1 + 2 + 3 + 4 + \ldots + 98 + 99 + 100$ was $50 \times 101 = 5050$.

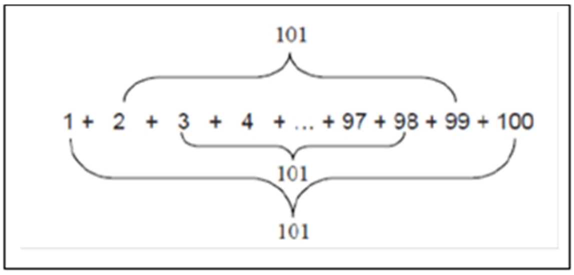


Figure 1. Addition of whole numbers from 1 to 100.

What can we learn from the young Gauss? The ability to see the structure or 'pattern' has implications of mathematics teaching and learning. Students could see the beauty of the pattern resulting from the reasoning, innovation, and thinking and know the importance of looking for the pattern. On the other words, teaching mathematics means helping students to find patterns and to apply them to solve problems. At the end, students are able to transfer their understanding of patterns to other areas and problems. For example, imagine that the problem was 1 + 2 + 3 + 4 + … + 998 + 999 + 1000. By using young Gauss' method, the sum was: ½ × (1000) × (1001) = 500.500. It is clear that the knowledge of pattern allows the transfer of knowledge.

De Lange (2005) stated that mathematics could be seen as the language that describes patterns – both patterns in nature and patterns invented by the human mind. The Marquis de Condorcet (in Fitzgerald and James, 2007, p. ix) stated: "Mathematics … is the best training for our abilities, as it develops both the power and the precision of our thinking."

Based on this brief discussion, some conclusions can be drawn as follows: (1) teacher should start the lesson with a task or problem; (2) teacher should give an opportunity to his/her students to solve the given task or problem; (3) there are a range of solutions to any task or problem; (4) the power of patterns often gives better, easier, and reasonable results; (5) the process of solving problem shows the importance of thinking, reasoning, and creativity; (6) students should be given opportunities to learn to think, to reason, and to be creative; (7) Gauss applied SA steps in solving the problem namely observing, questioning, collecting data, reasoning, as well as communicating; and (8) the power of patterns in mathematics can be achieved by using SA.

An Example of PSA and Its Interpretation Using SA

In Japan, Isoda and Katagiri (2012, p.31) stated that the general aims of education in Japan are:

> … to develop qualifications and competencies in each individual school child, including the ability to find issues by oneself, to learn by oneself, to think by oneself, to make decisions independently and to act. So that each child or student can solve problems more skilfully, regardless of how society might change in the future.

To ensure that mathematics teaching and learning in Japan focuses on problem-solving as mentioned above, Isoda and Katagiri (2012) proposed the PSA. The lesson is started by 'Problem Posing' as shown in Figure 2.

A three digit number is subtracted by a two digit number and the result must be 3. Study it.

-	-	-
–	-	-
		3

Figure 2. Example of problem in 'Posing Problem'.

In the second step, each student solves problem individually. Students might find several solutions as follow.

1	0	0	1	0	1	1	0	2
–	9	7	–	9	8	–	9	9
		3			3			3

Figure 3. Some solutions from students.

Next, 'Comparison and Discussion' is conducted. If students work shows that the minuends are 100, 101, and 102 (see Figure 3), the following patterns that can be drawn.

1. If the minuend is added/subtracted by 1, the subtrahend should be added/subtracted by 1. The patterns will be correct also if the minuend was added/subtracted by 2, and so on.
2. If the difference is 3, then students can only find 3 equations also.

Observing pattern shows the beauty of mathematics and encourages students' curiosity such as: Did the two patterns happen by coincidence? Can they be proved or verified? If the difference is 3, then why we can only find 3 equations also? What is the reason? What will happen if the difference is 5, instead of 3, can we find 5 equations also? Can we make a proof?

Those question should be answered by reasoning in the form of argument and proof. As presented in Figure 4, if the difference is 3, then three equations can be found, in which the minuends are 100, 101, and 101 while the subtrahends are 97, 98, and 99. If we apply the first pattern then the subtrahend of the previous equation is 96 and its minuend is 99, while the minuend of the next equation is 103 and its subtrahend is 100. In both cases, it contradicts the conditions that the minuend is a 3-digit number while its subtrahend is a 2-digit number. Thus, if the difference is 3, there will only 3 equations.

1	?	?	1	0	0	1	0	1	1	0	2	1	0	3
–	9	6	–	9	7	–	9	8	–	9	9	–	?	?
		3			3			3			3			3

Figure 4. The extended pattern.

Students can also collect more data by experimenting if the difference is 5 and they find 5 equations as shown in Figure 5.

$$100 - 95 = 5 \qquad 101 - 96 = 5 \qquad 102 - 97 = 5 \qquad 103 - 98 = 5 \qquad 104 - 99 = 5$$

Figure 5. The extended problem.

The above example from Japan can be used to show the relationship between PSA and SA. The PSA in the first step can support the SA from Indonesia by introducing problem posing and in the last step by extending the investigation to establish a proof or general pattern. The PSA and SA can be compared in this following modified table (Shadiq, 2017).

Table 1

Comparison of Steps of Japanese PSA and Indonesian SA

No	The PSA (Japanese)	No	The SA (Indonesian)
1.	Problem posing		
2.	Independent solving	1.	Observing
		2.	Questioning
		3.	Experimenting, collecting data
		4.	Reasoning
3.	Comparison and discussion	5.	Communicating
4.	Summary and integration		

Based on the table, it can be concluded that teaching and learning process should be started with a contextual problem which is in line with the first step of PSA to ensure that the SA can be observed during mathematics teaching and learning.

Example of SA by Using the Preferred Method

Isoda and Katagiri (2012) used equilateral triangle problem in explaining ‘the preferred method’. Before posing it to students, teacher is supposed to try to solve the problem first.

How many all of the equilateral triangles are there on the 4^{th} and 10^{th}?

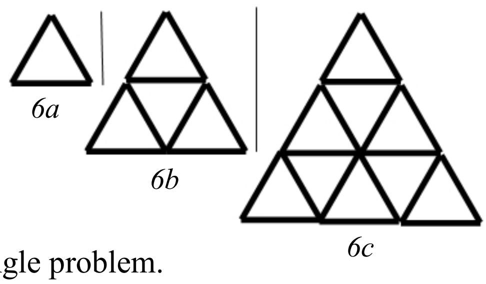
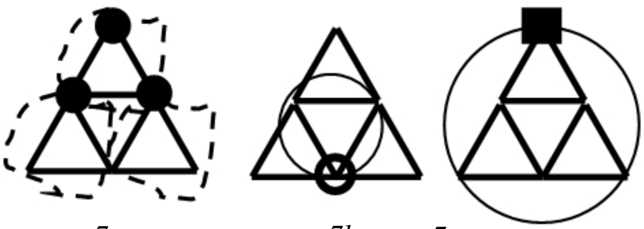


Figure 6. The equilateral triangle problem.

The steps of 'the preferred method' proposed by Isoda and Katagiri (2012, p.31) are as follows:

1. Clarification of the task part 1, students should find out all triangles from all dimensions of the triangles.
   It is clear that Figure 6a is made from 1 equilateral triangle only, while in Figure 6b there are 5 equilateral triangle.
   a. 3 equilateral triangle with 1 × 1 × 1 dimension facing up, such as '▪'.
   b. 1 equilateral triangle with 1 × 1 × 1 dimension facing down, such as '▪'.
   c. 1 equilateral triangle with 2 × 2 × 2 dimension facing up.



7a *7b* *7c*

Figure 7. The number of triangles from Figure 6b.

2. Clarification of the task part 2, teacher should remind students to think of the best ways of counting (better and easier). Facilitate students to learn to be systematic.
   These are some alternatives to find the best way of counting.
   a. In Figure 7a, there are 3 equilateral triangles with 1 × 1 × 1 dimension facing up, such as '▪'. Students can also use 3 closed curves or 3 dots representing those 3 equilateral triangle.
   b. In Figure 7b, there is 1 equilateral triangle with 1 × 1 × 1 dimension facing down, such as '▪'. Students can use 1 circle or 1 dot representing that 1 equilateral triangle.
   c. In Figure 7c, there is 1 equilateral triangle with 2 × 2 × 2 dimension facing up. Students can use 1 circle or 1 rectangle representing that 1 equilateral triangle.
3. Facilitate students to realize the benefit of sorting. In this case, the benefit of sorting can be experienced through sorting the dimension of the equilateral triangles, such as the dimension of 1 × 1 × 1, 2 × 2 × 2, 3 × 3 × 3, … , or based on the direction of the equilateral triangles, up or down, such as ▪ or ▪.
4. Facilitate students to learn the benefit of coding (naming), such as the dimension of 1× 1 × 1, 2 × 2 × 2, 3 × 3 × 3, … representing the side length of the equilateral triangles found by students or how the equilateral triangle facing up, such as '▪' or the equilateral triangle facing down, such as '▪'.
   From Figure 8a, ▪ABC is an equilateral triangle with 1 × 1 × 1 dimension and facing up, ▪BDC is an equilateral triangle with 1 × 1 × 1 dimension and facing down, and ▪AEF is equilateral triangles with 3 × 3 × 3 dimension and facing up. It will be very difficult to explain the difference among ▪ABC, ▪BDC, and ▪AEF if the new terms such as 'dimension', 'facing up', and 'facing down' are not yet introduced. Students are welcome to use other terms.

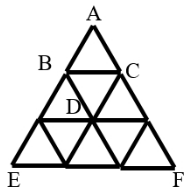


Figure 8a. Coding.

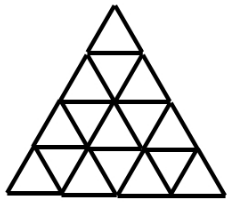


Figure 8b. The 4th figure.

5. Facilitate students to learn to validate the correctness or the reasonableness of the result. Referring to Figure 8b, teacher may ask: How many triangles are there in the 4th figure? The answer is 27. At least four alternatives can be identified as follows:
   Alternative 1, based on the number of equilateral triangle on each row and the dimension of the triangles. The number of equilateral triangles on each row, counting from the top, triangles with 1 × 1 × 1 dimension is (1 + 3 + 5+ 7); the number of equilateral triangles on each row with 2 × 2 × 2 dimension is (1 + 2 + 3 + 1); the number of equilateral triangles on each row with 3 × 3 × 3 dimension is (1 + 2); and the number of equilateral triangle with 4 × 4 × 4 dimension is (1). So, the number of all of equilateral triangles on the 4th figure is [(1 + 3 + 5 + 7)] + [(1 + 2 + 3 + 1)] + [(1 + 2)] + [(1)] = 16 + 7 + 3 + 1 = 27.
   Alternative 2, based on the number of equilateral triangle on each row, the dimension of the triangles, and the position of the triangles (facing up or facing down).
   The number of equilateral triangles facing up on each row, counting from the top, with 1 × 1 × 1 dimension is (1 + 2 + 3 + 4); the number of equilateral triangles facing up on each row, counting from the top, with 2 × 2 × 2 dimension is (1 + 2 + 3); the number of equilateral triangles facing up on each row, counting from the top, with 3 × 3 × 3 dimension is (1 + 2); and the number of equilateral triangle facing up on each row, counting from the top, with 4 × 4 × 4 dimension is (1). Then, the number of equilateral triangles facing down on each row, counting from the top, with 1 × 1 × 1 dimension is (1 + 2 + 3) and the number of equilateral triangles facing down with 2 × 2 × 2 dimension is (1). So, the number of all of equilateral triangles on the 4th figure is [(1 + 2 + 3 + 4) + (1 + 2 + 3) + (1 + 2) + (1)] + [(1 + 2 + 3) + (1)] = [10 + 6 + 3 + 1] + [6 + 1] = [20] + [7] = 27.
   Alternative 3, based on the position of the points or vertex of each triangle on the figure. The activity can be started from the top vertex, continuing with the vertices on the first vertical line and so on.

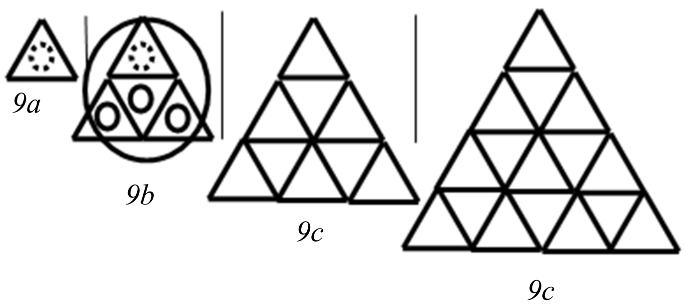


Figure 9. Alternative 4.

Alternative 4, simultaneously based on the number of equilateral triangles from each previous figure.
In Figure 9a there is only 1 equilateral triangle, while in Figure 9b there are 5 equilateral triangles. It can be notated that there are 1 + 4 = 5 equilateral triangles on the second figure. On the third figure, the number of equilateral triangles are [1] + [1 + 2 + 1] + [1 + 2 + 3 + 2] = 1 + 4 + 8 = 13. Finally, on the forth figure, the number of equilateral triangles are [1] + [1 + 2 + 1] + [1 +2 + 3 + 2] + [1 + 2 + 3 + 4 + 3 + 1] = 1 + 4 + 8 + 14 = 27.

Several patterns, in which students are expected to reason, can be identified as follows:

a. The number of equilateral triangles facing up with 1 × 1 × 1 dimension on n × n × n dimension figure is: (1 + 2 + 3 + 4 + … n). Why?
b. The number of equilateral triangles facing up with k × k × k dimension on n × n × n dimension figure is: (1 + 2 + 3 + 4 + … + (n ▪ k)). Why?
c. The number of equilateral triangles facing down with 1 × 1 × 1 dimension on n × n × n dimension figure is: (1 + 2 + 3 + 4 + … + (n ▪ 1)). Why?
d. The number of equilateral triangles facing up with k × k × k dimension on n × n × n dimension figure, in which k = ½ n, is 1. For example, if n = 4 and k = 2, then on Figure 4, with n = 4 there is only one equilateral triangle facing down with 2 × 2 × 2 dimension. However, on Figure 5, with n = 5 there are 1 + 2 = 3 equilateral triangle facing down with 2 × 2 × 2 dimension and no equilateral triangle facing down with 3 × 3 × 3 dimension. Why?
e. Facilitate students to learn to come up with a more accurate and convenient counting method. From the four alternative strategies that have been identified, with or without teacher's help, by using Alternative 2, hopefully students can decide that on the 10^{th} figure, the number of equilateral triangles will be:
[(1 + 2 + 3 +…+ 10) + (1 + 2 + 3 +…+ 9) + (1 + 2 +…+ 8) + (1 + 2 +…+ 7) + … + [(1 + 2 + 3 + 4) + (1 + 2 + 3) + (1 + 2) + (1)] + [(1 + 2 + 3 + 4 + 5)+ [(1 + 2 + 3) + (1)].

From the explanation above, in mathematics problem-solving, the emphasis should be given to the importance of pattern to produce better, easier, and reasonable solutions. To ensure this, mathematics teachers should start the activity with the task or problem. Later, mathematics teachers should give students opportunities to solve problems independently to ensure they develop their skills and knowledge to solve daily problems.

Teacher as Facilitator

It cannot be denied the crucial roles of mathematics teachers in facilitating and helping their students. Even and Ball (2009, p.1) stated: "... teachers are key to students' opportunities to learn mathematics." Ki Hadjar Dewantara, one of Indonesian education leading figures in 1900s (Kemdikbud, 2011, p. 28) stated: *ing ngarsa sung tuladha, ing madya mangun karsa, tut wuri handayani* (in front [a teacher] should set an example, in the middle [a teacher] should raise the spirit, behind [a teacher] should give encouragement) to describe how an ideal teacher supposed to be. The last part of the maxim *Tut Wuri Handayani* is used as the motto of Indonesian MoEC.

Conclusion

Australia, through the Australian Association of Mathematics Teachers (AAMT, 2013, p. 1), stated:

> Highly successful teachers of Mathematics are active, lifelong learners. They engage in professional learning processes that include collegial interaction, professional reading and active exploration of new teaching ideas, practices and resources in the classroom. They reflect on practice and their new knowledge they gain, and learn from their experiences. Their active participation is based on their commitment to attaining the best possible outcomes for all learners and to help build the capacity of other educators.

To be an 'experienced' teacher, a teacher should prepare the lesson by doing task or problem to be given to her/his students, finding out all the alternative solutions, anticipating students' difficulties and questions, and helping them to reflect on what they are doing during the teaching and learning process. White (2011, p. 9) stated:

> Teaching is a process of continual striving for excellence, a quest for the perfect lesson and an understanding that it can never be achieved. There is something, upon reflection, that could be improved to meet the individual needs of the students. It is the combination of reflection, professional learning and experience that contributes to the gradual accumulation of pedagogical knowledge and super power.

To put this in an Indonesian context, Indonesian mathematics teachers should stay in front of their students as an example or as a model (*ing ngarsa sung tuladha*). On the other word, teachers should choose the most suitable tasks, problems, or activities (contextual, realistic or mathematical) for students and guide them through the learning process. Next, they should stay in the middle of their students to raise their spirit (*ing madya mangun karsa*). In this part, teachers are expected to praise students' effort in completing the task or problem as well as provide students with scaffolding. At last, Indonesian teachers must encourage their students develop their potential and capabilities (*tut wuri handayani*) by keeping motivating them.

These expectations requires some changes. The Indonesian curriculum, delivery system, and assessment in pre-service and in-service institution should be evaluated to meet the need of the young generation.

1. Further research should be carried out to assess and enhance the ability of mathematics teachers in problem-solving, exploring, investigating, experimenting, discovering, and inquiring.
2. The nature and practice of national examination should focus not only on content knowledge but also on process of acquiring skills and knowledge to ensure that mathematics teaching and learning in the classroom is able to help students to be independent learners.

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